Communication under Language Barriers*

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Abstract

We study the welfare effect of language barriers in communication. Specifically, we compare the equilibrium welfare in a game with language barriers to that in the equivalent game without language barriers. We show how and why language barriers may (weakly) improve welfare by providing two positive results. First, in a game with *any* language barriers, we prove that if we allow for N-dimensional communication, *any* equilibrium outcome of the equivalent game without language barriers can be replicated. Second, for *any* payoff primitive, we provide a welfare ranking for several noisy-communication devices, including language barriers, that generalizes the results in Goltsman, Hörner, Pavlov, and Squintani (2009). In particular, our results imply that there always exist *some* language barriers whose maximal equilibrium welfare (always weakly and sometimes strictly) dominates *any* noisy-talk equilibrium (and hence also any cheap-talk equilibrium) under no language barriers.

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1 Introduction

Communication is often about transmission of information, so that a natural question to ask is "what" information is actually transmitted and this has been the focus of the literature on strategic communication, or "cheap talk." This literature, however, has typically ignored the issue of "how" information is transmitted. Yet, everyday experience suggests that how information is transmitted may both hinder or help communication. For instance, it is notoriously hard to convey humor or any other emotion in modern electronic communication, and emoticons were developed as a response to the problem (Curran and Casey (2006)). Similarly, there is a concern that patients may not be able to understand medical jargon, so that the common recommendation is not to release to patients their medical records or at least avoid jargon when this is likely to cause misunderstandings (see Ross and Lin (2003) for a survey of the medical literature on this issue).[1] In this paper, we take the "how" issue seriously, and study a model with "language types," as introduced by Blume and Board (2013), which allows us to model the possibility of "language barriers" within a strategic communication setting.

In the canonical Crawford and Sobel (1982) sender-receiver setting, it is implicitly assumed that all participants have perfect language ability. In contrast to this standard framework, Blume and Board (2013) introduce "language types" for both sender and receiver which describe, respectively, the sets of messages that can be sent and that can be understood.[2] This provides a parsimonious way to study a fundamental question: do language barriers improve or harm welfare or, equivalently, is equilibrium welfare under language barriers greater or smaller than that under no language barriers? We pursue this fundamental question in two directions. We first ask: is there a communication protocol that can guarantee that *any* language barriers won't impact negatively on welfare? We then ask: for *any* payoff primitive, can we find *some* language barriers that (weakly)

[1] Additional examples are discussed in Blume and Board (2013) and Blume (2018). We should also point out that the literature on organizations has paid significant attention to the "how" issue in communication, at least since Arrow (1975). Garicano and Prat (2013) provide a survey of the recent literature.

[2] When we say a receiver's language type represents the set of messages she understands, we mean that she can distinguish all messages in such a set and thus condition her actions on each of those messages. However, she cannot distinguish any two messages that do not belong to her language type (they all appear to be the same "nonsense" to her). Therefore, she must choose the same action in response to any such "nonsense" (see details in Section 2).

improve equilibrium welfare, even if we stick to the canonical communication protocol? We provide positive answers to both questions.

Our first main result is inspired by a phenomenon that we observe in real-life communication, which is that messages are formed by combining basic units to make complex structures that convey meaning. Thus, it seems restrictive to assume a fixed number of messages in modeling communication, each message with a predetermined level of complexity, rather than assuming that such messages can always be used as building blocks capable of forming more sophisticated structures. The communication protocol in Blume and Board (2013) *implicitly* forbids forming more complex structures than the 1-dimensional messages in a set M, so we relax this assumption in the simplest way possible by assuming that the set of available messages extends to M^N (for some integer N). Our first main result is that, under some minimal assumptions, any equilibrium which would obtain in a game with 1-dimensional communication and no language barriers can be replicated by an equilibrium of the same game if we add any language barriers (independent of payoff states) but allow for N-dimensional communication (for sufficiently large N), i.e., language barriers do not harm welfare under multi-dimensional communication.

To achieve this result, we need to overcome three technical difficulties under language barriers: (1) the sender may not know the receiver's language type; (2) the receiver may not know the sender's language type; (3) there may not be enough common messages (between sender and receiver) to transmit useful information. It is straightforward to see that multi-dimensional communication overcomes the third difficulty, because the set of common messages expands as we increase the dimension of messages. The novelty of our equilibrium construction comes from how it overcomes the first two difficulties, although it does so in a way that resembles real-life solutions to similar problems. In particular, in our equilibrium construction, a sender partitions her N-dimensional message into several blocks of sub-messages, with each block intended for a specific language type of the receiver. We show that this overcomes the first difficulty just as businesses - which sell the same product in different countries - solve the problem of communicating with customers who speak different languages by producing an instruction manual with the same instructions written in all the relevant languages. In addition, in our construction, each block of sub-messages from the sender is further divided into two parts, with one part voluntarily revealing the sender's language type, and the other part transmitting payoff-relevant information (using type-specific common messages). The part where the sender's language type is revealed overcomes the second difficulty, just as people sometimes add a "smiley face" emoticon in an email message to ensure the content is not taken too seriously.[3]

In the second part of the paper, we tackle the second question in the context of 1-dimensional communication. In particular, we compare welfare across several protocols for cheap-talk communication. Our main result is a linear ranking of the maximal welfare achieved in these different protocols:

$$\Phi^{LB} \geq \Phi^{M} \geq \Phi^{ILB} \geq \Phi^{N}$$

where Φ^{LB}, Φ^{M}, Φ^{ILB}, Φ^{N} are the maximal equilibrium welfare achieved in a generic sender-receiver game under language barriers, mediation, language barriers with the restriction that language types are distributed independently of payoff states (we refer to these as independent language barriers from now on), and noisy talk, respectively. Both Goltsman, Hörner, Pavlov, and Squintani (2009) and Blume and Board (2010) ask a similar question assuming quadratic preferences and the uniform payoff distribution, and (together) establish the welfare equivalence result, $\Phi^{M} = \Phi^{ILB} = \Phi^{N}$. Instead, we show that equilibria with language barriers, mediation, independent language barriers and noisy talk correspond to a series of increasingly restrictive incentive compatibility conditions (in that order), which generate the welfare order described above. Thus, our results go beyond the environment with quadratic preferences and the uniform payoff distribution and indeed hold for *any* general preference and distributional assumptions. This greater generality allows us to show, through an example (Example 2A available in Giovannoni and Xiong (2018)), that in some environments $\Phi^{ILB} > \Phi^{N}$, thus breaking the welfare equivalence established by Goltsman, Hörner, Pavlov, and Squintani (2009) and Blume and Board (2010).

The remainder of the paper proceeds as follows: Section 2 describes the model; Section 3 studies N-dimensional communication; Section 4 focuses on 1-dimensional communication; Section 5 reviews the related literature; Section 6 concludes.

[3] Or, equivalently, the sender uses a part of her message - the "smiley face" - to identify herself as the "humorous" type and not the "serious" type.

2 Model

Following Crawford and Sobel (1982) and Blume and Board (2013), we consider a sender-receiver model of communication throughout the paper. That is, $I = \{1,2\}$ denotes the set of players, where player 1 is a sender and player 2 is a receiver. Player 1 privately observes a payoff state $t \in T$, and player 2 takes a payoff-relevant action $a \in A$. Every agent $i \in I$ has a utility function $u_i : T \times A \longrightarrow \mathbb{R}$. Throughout the paper, a subscript i refers to agent i, whereas no subscript refers to all agents.

Let M denote the set of possible messages. For simplicity, assume $T \cup M \cup A \subset \mathbb{R}$. For every $i \in I$, we use a non-empty $\Lambda_i \subset 2^M \diagdown \{\varnothing\}$ to denote the set of language types of agent i. Each language type $\lambda_i \in \Lambda_i$ is defined as the set of messages that agent i understands. Define $\Lambda = \Lambda_1 \times \Lambda_2$. There is a common prior $\pi \in \triangle(T \times \Lambda)$, and π_T and π_Λ denote the corresponding marginal distributions. Let $\pi(\cdot\,|t,\lambda_1)$ and $\pi(\cdot\,|\lambda_2)$ denote the distributions of λ_2 and (t,λ_1) conditional on (t,λ_1) and λ_2, respectively. We will sometimes impose the following assumption, and we will state it explicitly if we do.

Assumption 1 *$t \in T$ and $\lambda \in \Lambda$ are independently distributed under π.*

We use $|X|$ to denote the cardinality of a set X. Throughout the paper, we assume $|M| > 1$ and $|\Lambda| < \infty$. As usual, $-i$ represents $I \diagdown \{i\}$, and x_{-i} represents $(x_j)_{j \in I \diagdown \{i\}}$.

For a positive integer N, we define an N-dimensional communication game. Before the game starts, nature chooses a state-type profile (t,λ) according to π. Then, upon privately observing $(t,\lambda_1) \in T \times \Lambda_1$, player 1 sends an N-dimensional message $m \in (\lambda_1)^N$ to player 2. Finally, upon privately observing $[\lambda_2, m]$, player 2 takes an action $a \in A$.

Thus, a game is defined by a tuple $\langle M, T, \Lambda, \pi, A, (u_i : T \times A \longrightarrow \mathbb{R})_{i \in I}, N \rangle$, and players' strategies in the game are

$$\begin{aligned} \text{player 1} &: \quad \sigma : T \times \Lambda_1 \to M^N, \\ \text{player 2} &: \quad \rho : \Lambda_2 \times M^N \to A, \end{aligned}$$

such that σ and ρ are *regular* with respect to Λ_1 and Λ_2, respectively.[4] Regularity for σ

[4]For notational ease, we focus on pure strategies. The analysis can be easily extended to mixed strategies but at the cost of significantly more notation.

means that $\sigma(t,\lambda_1) \in (\lambda_1)^N$ for every $(t,\lambda_1) \in T \times \Lambda_1$. The interpretation is that a language type λ_1 of the sender understands only the messages with which she is endowed, and hence this type can only send a vector message where each component is in λ_1. This restriction defines "language barriers" for the sender.

We define the regularity for ρ as follows. For any $x, y \in M$, we say that $x \sim_{\lambda_2} y$ if and only if $x = y \in \lambda_2$ or $\{x, y\} \cap \lambda_2 = \varnothing$. That is, type λ_2 can distinguish any two messages in λ_2, but treats all the other messages as a single and distinct "nonsense" message. Then, for any positive integer K, and any $x = (x^1, ..., x^K)$, $y = (y^1, ..., y^K) \in M^K$, we say that $x \sim_{\lambda_2} y$ if and only if $x^k \sim_{\lambda_2} y^k$, $\forall k \in \{1, ..., K\}$.[5] Then, regularity of ρ requires

$$m \sim_{\lambda_2} m' \Longrightarrow \rho[\lambda_2, m] = \rho[\lambda_2, m'], \qquad (1)$$
$$\forall (\lambda_2, m, m') \in \Lambda_2 \times M^N \times M^N.$$

(1) says that the receiver's strategy must be measurable with respect to his language type, which captures "language barriers" for the receiver.

Given a strategy profile (σ, ρ) and a state-type profile (t, λ), define

$$U_i(\sigma, \rho | t, \lambda) = u_i(t, \rho[\lambda_2, \sigma(t, \lambda_1)]), \forall i \in I,$$

i.e., $U_i(\sigma, \rho | t, \lambda)$ is the final utility of agent i given $[(\sigma, \rho), (t, \lambda)]$. We adopt the solution concept defined as follows.

Definition 1 *(σ, ρ) is an equilibrium if*

$$\int_{\Lambda_2} [U_1(\sigma, \rho \mid t, \lambda) - U_1(\sigma', \rho \mid t, \lambda)] \pi(d\lambda_2 | t, \lambda_1) \geq 0, \forall (t, \lambda_1) \in T \times \Lambda_1, \forall \sigma', \qquad (2)$$

$$\textit{and} \int_{T \times \Lambda_1} [U_2(\sigma, \rho \mid t, \lambda) - U_2(\sigma, \rho' \mid t, \lambda)] \pi[d(t, \lambda_1) | \lambda_2] \geq 0, \forall \lambda_2 \in \Lambda_2, \forall \rho'. \qquad (3)$$

I.e., we adopt the standard notion of Bayesian Nash equilibrium applied to this specific setup, where (2) and (3) describe incentive compatibility conditions for the players. Throughout the paper, we impose the following necessary assumptions for informative communication:[6]

$$|\lambda_1| \geq 2, \forall \lambda_1 \in \Lambda_1, \qquad (4)$$
$$\lambda_1 \cap \lambda_2 \neq \varnothing, \forall (\lambda_1, \lambda_2) \in \Lambda. \qquad (5)$$

[5] We use "$x \nsim_{\lambda_2} y$" to denote that "$x \sim_{\lambda_2} y$" is false.

[6] If (4) is violated, we have $|\lambda_1| = 1$, i.e., sender λ_1 always sends the same message, which is not informative. If (5) is violated, receiver λ_2 always gets the "nonsense" message from λ_1, which is not informative.

3 Main Results: N-dimensional Communication

We study N-dimensional communication in this section, and prove that any equilibrium in a communication game with no language barriers can be replicated by an equilibrium of the corresponding game with language barriers as long as we allow for messages of a sufficiently high dimension. In Section 3.1, we first define what this means formally, while in Section 3.2, we state our main result, discuss the intuition behind our construction and provide a proof.

3.1 Similar games and outcome-equivalent equilibria

We will compare equilibria between communication games which differ only on language barriers. To make the comparison between two such games meaningful, they must be "similar", which means that they must share the same primitives (actions, payoff states, etc.), but may differ in language types and the dimension of messages they send.

Definition 2 *Two games $\widehat{G}$ and $\overline{G}$*

$$\begin{aligned}\widehat{G} &= \left\langle \widehat{M}, \widehat{T}, \widehat{\Lambda}, \widehat{\pi}, \widehat{A}, \left(\widehat{u}_i : \widehat{T} \times \widehat{A} \longrightarrow \mathbb{R}\right)_{i\in I}, \widehat{N} \right\rangle, \\ \overline{G} &= \left\langle \overline{M}, \overline{T}, \overline{\Lambda}, \overline{\pi}, \overline{A}, \left(\overline{u}_i : \overline{T} \times \overline{A} \longrightarrow \mathbb{R}\right)_{i\in I}, \overline{N} \right\rangle,\end{aligned}$$

are similar, denoted by "$\widehat{G} \sim \overline{G}$", if

$$\left\langle \widehat{M}, \widehat{T}, \widehat{A}, (\widehat{u}_i)_{i\in I} \right\rangle = \left\langle \overline{M}, \overline{T}, \overline{A}, (\overline{u}_i)_{i\in I} \right\rangle \text{ and } \widehat{\pi}_{\widehat{T}} = \overline{\pi}_{\overline{T}}.$$

We now define outcome-equivalent equilibria in two similar games.

Definition 3 *Given two similar games, $\widehat{G}$ and $\overline{G}$, an equilibrium $(\widehat{\sigma}, \widehat{\rho})$ in $\widehat{G}$ is outcome-equivalent to an equilibrium $(\overline{\sigma}, \overline{\rho})$ in $\overline{G}$ if*

$$\widehat{\rho}\left[\widehat{\lambda}_2,\, \widehat{\sigma}\left(t,\, \widehat{\lambda}_1\right)\right] = \overline{\rho}\left[\overline{\lambda}_2,\, \overline{\sigma}\left(t,\, \overline{\lambda}_1\right)\right], \forall \left(t, \widehat{\lambda}, \overline{\lambda}\right) \in T \times \widehat{\Lambda} \times \overline{\Lambda}.$$

Outcome-equivalent equilibria in similar games induce the same action for any given payoff state, regardless of language types. As a result, they induce the same *ex-post* utility for every player.[7]

3.2 Outcome-equivalence for similar games

Let $\mathcal{G}^*$ denote the set of all standard communication games with 1-dimensional messages and no language barriers, i.e.,

$$\mathcal{G}^* \equiv \left\{ \left\langle M, T, \Lambda^*, \pi, A, (u_i : T \times A \longrightarrow \mathbb{R})_{i \in I}, N^* \right\rangle : \begin{array}{c} \lambda_i^* \equiv M, \Lambda_i^* \equiv \{\lambda_i^*\}, \\ \Lambda^* = \prod_{i \in I} \Lambda_i^*, N^* \equiv 1 \end{array} \right\}.$$

For any $\widetilde{\Lambda}$ and any positive integer $\widetilde{N}$, define

$$\mathcal{G}^{(\widetilde{\Lambda}, \widetilde{N})} \equiv \left\{ \langle M, T, \Lambda, \pi, A, (u_i : T \times A \longrightarrow \mathbb{R})_{i \in I}, N \rangle : (\Lambda, N) = \left(\widetilde{\Lambda}, \widetilde{N}\right) \right\},$$

i.e., $\mathcal{G}^{(\widetilde{\Lambda}, \widetilde{N})}$ contains all $\widetilde{N}$-dimensional communication games with language structure $\widetilde{\Lambda}$.

For an equilibrium (σ, ρ) in a game $\langle M, T, \Lambda, \pi, A, (u_i : T \times A \longrightarrow \mathbb{R})_{i \in I}, N \rangle$, define

$$\mathcal{E}^{(\sigma, \rho)} \equiv \{\sigma(t, \lambda_1) : t \in T \text{ and } \lambda_1 \in \Lambda_1\},$$

i.e., $\mathcal{E}^{(\sigma, \rho)}$ is the set of messages player 1 sends in the equilibrium. We say (σ, ρ) is a finite-message equilibrium if $\left|\mathcal{E}^{(\sigma^*, \rho^*)}\right| < \infty$ and an infinite-message equilibrium otherwise. For simplicity, we focus on finite-message equilibria here, but our analysis can be easily extended to infinite-message equilibria, as is done in Appendix B.2.

Theorem 1 *Given Assumption 1, for any Λ and any finite-message equilibrium (σ^*, ρ^*) in a game $G^* \in \mathcal{G}^*$, a positive integer $\mathcal{N}$ exists, such that in any game $G \in \cup_{N \geq \mathcal{N}} \mathcal{G}^{(\Lambda, N)}$ with $G \sim G^*$, there exists an equilibrium (σ, ρ) of G that is outcome-equivalent to (σ^*, ρ^*).*

We now proceed to first provide some intuition for our multi-dimensional construction behind Theorem 1 and then provide a proof. In Appendix B.1 we also discuss its application to common-interest games.

[7] In a sense, our positive result based on this ex-post outcome equivalence notion implies that language types do not matter at all. One could also adopt a weaker notion of outcome-equivalence: the induced joint distribution on payoff types and actions are the same. Clearly, our main result would still hold under this weaker notion.

3.2.1 The role of N-dimensionality in Theorem 1

To guarantee effective communication, we need to tackle three difficulties: (1) the sender may not know the receiver's language type; (2) the receiver may not know the sender's language type; (3) there may not be enough common messages between sender and receiver to transmit information. In this section, we leave incentive compatibility aside, and show that sufficiently many dimensions enable players to tackle all of the difficulties. We return to incentive compatibility in the next section, and prove Theorem 1 by showing that players are indeed willing to utilize such abilities to achieve effective communication.

First difficulty: the receiver's language type is his private information. The sender can partition a N-dimensional message into $|\Lambda_2|$ blocks, with each block intended for a language type of the receiver. Specifically, suppose $N = N' \times |\Lambda_2|$ for some integer N', and a sender's N-dimensional message is $m = \left(m^{\lambda_2}\right)_{\lambda_2 \in \Lambda_2} \in \left[M^{N'}\right]^{|\Lambda_2|}$, where $m^{\lambda_2} \in M^{N'}$ is the intended message from the sender to λ_2. Upon receiving m, the language type λ_2 just goes to his designated block to retrieve his intended message m^{λ_2}. Thus, messages with multiple dimensions do more than just increase the size of the message space. As an example, consider the following common-interest game. Suppose that $T = A = \{\alpha, \beta, \gamma, \delta\}$ and $u_1(t,a) = u_2(t,a) = 1$ if $a = t$ and zero otherwise. Let

$$M = \mathbb{Z};\ \Lambda_1 = \{\lambda_1\};\ \Lambda_2 = \left\{\lambda_2^-, \lambda_2^+\right\};$$
$$\lambda_1 = \mathbb{Z};\ \lambda_2^- = \{-1, -2, ...\};\ \lambda_2^+ = \{1, 2, ...\};$$

Suppose every $(t, \lambda_1, \lambda_2) \in T \times \Lambda_1 \times \Lambda_2$ has positive probability. Since the sender and the receiver have identical preferences, the only issue is how the sender can communicate her information to the receiver. Clearly, without language barriers, the efficient outcome (i.e., full communication) is an equilibrium. An efficient equilibrium under these language barriers requires that both λ_2^- and λ_2^+ be able to distinguish between each of the equilibrium messages from the sender in each of the states. But for this to occur, at least three of the four messages must be in λ_2^-, and at least three of the four must be in λ_2^+, which is impossible. It is worth noting that $\left|\lambda_1 \cap \lambda_2^-\right| = \left|\lambda_1 \cap \lambda_2^+\right| = |\mathbb{Z}|$, i.e., the sender and the receiver always share infinitely-many common messages, but full communication still fails. However, we can achieve full communication even if we restrict λ_1 to the set $\{-3, -2, -1, 1, 2, 3\}$, but allow for 2-dimensional messages. Then, the sender can produce a message $(m^-(t), m^+(t))$ where m^- and m^+ are the blocks that describe the payoff relevant information for λ_2^- and λ_2^+, respectively.[8] Thus, the ability to use two-dimensional messages (rather than the number of common messages) is the key to improve communication.

Second difficulty: the sender's language type is her private information. Consider the following lemma, which we prove in Appendix A.1:

Lemma 1 *For any $\lambda_2 \in \Lambda_2$ and any*

$$\overline{N} > 3 + |\Lambda_1|, \tag{6}$$

there exists a function $\Upsilon_{\lambda_2} : \Lambda_1 \longrightarrow M^{\overline{N}}$ such that

$$\Upsilon_{\lambda_2}[\lambda_1] \in (\lambda_1)^{\overline{N}}, \forall \lambda_1 \in \Lambda_1,$$
$$\text{and } \lambda_1' \neq \lambda_1'' \implies \Upsilon_{\lambda_2}[\lambda_1'] \nsim_{\lambda_2} \Upsilon_{\lambda_2}[\lambda_1'']. \tag{7}$$

Suppose the sender follows Υ_{λ_2} in Lemma 1 to reveal her language type to type λ_2 of the receiver: if the sender is of type λ_1, she sends $\Upsilon_{\lambda_2}[\lambda_1] \in (\lambda_1)^{\overline{N}}$ to type λ_2. For any two distinct language types, λ_1' and λ_1'', (7) implies that λ_2 can distinguish the messages, $\Upsilon_{\lambda_2}[\lambda_1']$ and $\Upsilon_{\lambda_2}[\lambda_1'']$, sent by λ_1' and λ_1'', respectively, i.e., the sender's language types are fully revealed. To see the idea, consider the following example where now it is the receiver that is uncertain about the sender's language type. Here $T = A = \{\alpha, \beta\}$ and $u_1(t,a) = u_2(t,a) = 1$ if $a = t$ and zero otherwise. Let

$$M = \mathbb{Z};\ \Lambda_2 = \{\lambda_2\};\ \Lambda_1 = \{\lambda_1', \lambda_1'', \lambda_1'''\}$$
$$\lambda_2 = \{1,2,3\};\ \lambda_1' = \{1,2\};\ \lambda_1'' = \{2,3\};\ \lambda_1''' = \{1,3\}$$

Suppose every $(t, \lambda_1, \lambda_2) \in T \times \Lambda_1 \times \Lambda_2$ has positive probability. As in the previous example, without language barriers the efficient (full-communication) outcome is an equilibrium but, again, this does not hold for these particular language barriers. To see this,

[8] For instance, there is an equilibrium where the blocks m^- and m^+ are described by

$$m^-(\alpha) = -1,\ m^-(\beta) = -2,\ m^-(\gamma) = -3, m^+(\delta) = 1,$$
$$m^+(\alpha) = 1,\ m^+(\beta) = 2,\ m^+(\gamma) = 3, m^+(\delta) = -1,$$

i.e., type λ_2^- can understand $m^-(\alpha)$, $m^-(\beta)$ and $m^-(\gamma)$, while type λ_2^+ can understand $m^+(\alpha)$, $m^+(\beta)$ and $m^+(\gamma)$. For both language types, the message they cannot understand corresponds to δ.

suppose otherwise. Then, to achieve efficiency for λ_1', states α and β must be truthfully revealed by messages 1 and 2. Without loss of generality, suppose λ_2 plays α and β upon receiving messages 1 and 2, respectively. As a result, if λ_2 plays α upon receiving message 3, then efficiency is not achieved for λ_1'''; if λ_2 plays β upon receiving message 3, then efficiency is not achieved for λ_1'' - we get a contradiction. Nevertheless, because of Lemma 1, an equilibrium with multi-dimensional messages $\left(m^{\lambda}, m^{t}\left(\lambda\right)\right)$ inducing full communication, exists.[9] In such an equilibrium, the first component, m^{λ} identifies the sender's language type while the second component identifies the payoff state. As in the previous example, giving arbitrary additional messages to each sender type would not work because the receiver would not be able to understand such messages.

Final difficulty: not enough common messages. Once asymmetry of information about language types is resolved, the following lemma, which we prove in Appendix A.2, helps us resolve the final difficulty.

Lemma 2 *For any* $(\lambda_1, \lambda_2) \in \left[2^M \diagdown \{\varnothing\}\right]^2$ *and any finite-message equilibrium* (σ^*, ρ^*) *in a game* $G^* \in \mathcal{G}^*$ *and any*

$$\widehat{N} > \left|\mathcal{E}^{(\sigma^*,\rho^*)}\right|, \tag{8}$$

there exists a function $\Gamma_{(\lambda_1,\lambda_2)} : \mathcal{E}^{(\sigma^*,\rho^*)} \longrightarrow (\lambda_1)^{\widehat{N}}$ *such that for any* $m, m' \in \mathcal{E}^{(\sigma^*,\rho^*)}$,

$$m \neq m' \implies \Gamma_{(\lambda_1,\lambda_2)}\left(m\right) \nsim_{\lambda_2} \Gamma_{(\lambda_1,\lambda_2)}\left(m'\right). \tag{9}$$

By the previous two steps, both the sender's and the receiver's language types can be truthfully revealed. Given this, suppose sender λ_1 follows $\Gamma_{(\lambda_1,\lambda_2)}$ to send messages to receiver λ_2, and $\Gamma_{(\lambda_1,\lambda_2)}$ translates equilibrium messages in $\mathcal{E}^{(\sigma^*,\rho^*)}$ to i's endowed messages in $(\lambda_1)^{\widehat{N}}$. Then, for any two distinct messages m, m' in $\mathcal{E}^{(\sigma^*,\rho^*)}$, because of (9), receiver λ_2 can distinguish the two translated messages, $\Gamma_{(\lambda_1,\lambda_2)}\left(m\right)$ and $\Gamma_{(\lambda_1,\lambda_2)}\left(m'\right)$. That is, equilibrium messages are effectively transmitted.

[9] For instance, the following equilibrium achieves full communication:

$$\left(\begin{array}{c} \text{sender's strategy:} \left[\begin{array}{cc} \left(m^{\lambda_1'}, m^{\alpha}\left(\lambda_1'\right)\right) = (1,1), & \left(m^{\lambda_1'}, m^{\beta}\left(\lambda_1'\right)\right) = (1,2), \\ \left(m^{\lambda_1''}, m^{\alpha}\left(\lambda_1''\right)\right) = (2,2), & \left(m^{\lambda_1''}, m^{\beta}\left(\lambda_1''\right)\right) = (2,3), \\ \left(m^{\lambda_1'''}, m^{\alpha}\left(\lambda_1'''\right)\right) = (3,3), & \left(m^{\lambda_1'''}, m^{\beta}\left(\lambda_1'''\right)\right) = (3,1), \end{array}\right] \\ \text{receiver's strategy: choose } \alpha \text{ if messages in the two dimensions match, and } \beta \text{ otherwise.} \end{array}\right)$$

3.2.2 Proof of Theorem 1

Fix any game G^* without language barriers and any finite-message equilibrium (σ^*, ρ^*) in G^*. Consider $\mathcal{N} = \left(\overline{N} + \widehat{N}\right) \times |\Lambda_2|$, where $\overline{N}$ and $\widehat{N}$ are defined in (6) and (8), respectively. We now define a strategy profile (σ, ρ), and show it is an equilibrium in the $\mathcal{N}$-dimensional communication game which is similar to G^*.

The sender's strategy: let m_{λ_2} denote the message intended from the sender to type λ_2 of the receiver. For every $(t, \lambda_1) \in T \times \Lambda_1$, define

$$\sigma(t, \lambda_1) = (m_{\lambda_2})_{\lambda_2 \in \Lambda_2} = \left(\Upsilon_{\lambda_2}[\lambda_1], \Gamma_{(\lambda_1, \lambda_2)}[\sigma^*(t)]\right)_{\lambda_2 \in \Lambda_2} \in M^{\mathcal{N}}. \tag{10}$$

I.e., type λ_1 of the sender tells type λ_2 of the receiver about the sender's true language type via $\Upsilon_{\lambda_2}[\lambda_1]$ as described in Lemma 1 and the equilibrium message $\sigma^*(t)$ under (σ^*, ρ^*) via $\Gamma_{(\lambda_1,\lambda_2)}[\sigma^*(t)]$ as described in Lemma 2.

The receiver's strategy: fix any $\widetilde{t} \in T$. Upon receiving the intended message m_{λ_2} from the sender, type λ_2 of the receiver uses the following function to translate it back to an equilibrium message under (σ^*, ρ^*):

$$\Sigma_{\lambda_2}(m_{\lambda_2}) = \begin{cases} \sigma^*(t), & \begin{array}{l}\text{if there exists } (t, \lambda_1) \in T \times \Lambda_1 \text{ such that} \\ m_{\lambda_2} = \left(\Upsilon_{\lambda_2}[\lambda_1], \Gamma_{(\lambda_1,\lambda_2)}[\sigma^*(t)]\right);\end{array} \\ \sigma^*(\widetilde{t}), & \text{otherwise,} \end{cases} \tag{11}$$

where $\widetilde{t}$ is fixed above. Note that, by Lemmas 1 and 2, if there exist multiple $(t, \lambda_1) \in T \times \Lambda_1$ such that $m_{\lambda_2} = \left(\Upsilon_{\lambda_2}[\lambda_1], \Gamma_{(\lambda_1,\lambda_2)}[\sigma^*(t)]\right)$, then λ_1 must be unique, and all of such t must have the same equilibrium message $\sigma^*(t)$, so that $\Sigma_{\lambda_2}(m_{\lambda_2})$ is well-defined. Also, type λ_2 of the receiver *always* translates any m_{λ_2} back to *some equilibrium message* under (σ^*, ρ^*):

$$\text{for any } (\lambda_2, m_{\lambda_2}), \Sigma_{\lambda_2}(m_{\lambda_2}) = \sigma^*(t') \text{ for some } t' \in T, \tag{12}$$

We are ready to define ρ. For any $m = (m_{\lambda_2})_{\lambda_2 \in \Lambda_2} \in M^{\mathcal{N}}$, where m_{λ_2} is the message from the sender to type λ_2 of the receiver, we have

$$\rho(\lambda_2, m) = \rho^*\left[\Sigma_{\lambda_2}(m_{\lambda_2})\right], \forall \lambda_2 \in \Lambda_2. \tag{13}$$

Given the receiver playing ρ, *any* message of the sender would induce *some equilibrium action* under (σ^*, ρ^*). As immediately implied by (12) and (13),

$$\text{for any } (\lambda_2, m) \in \Lambda_2 \times M^{\mathcal{N}}, \rho(\lambda_2, m) = \rho^*\left[\sigma^*\left(t'\right)\right] \text{ for some } t' \in T, \tag{14}$$

To sum up, under (σ, ρ) and any given $(t, \lambda_1) \in T \times \Lambda_1$, each sender's type λ_1 follows $\sigma^*(t)$ by sending two pieces of information, $\Upsilon_{\lambda_2}[\lambda_1]$ and $\Gamma_{(\lambda_1,\lambda_2)}[\sigma^*(t)]$, to each receiver's type λ_2, where the former truthfully reveals λ_1, and the latter is the message $\sigma^*(t)$ coded via $\Gamma_{(\lambda_1,\lambda_2)}$ by using the messages available to λ_1. Upon receiving the message, each receiver λ_2 decodes it back to $\sigma^*(t)$, and plays the action $\rho^*[\sigma^*(t)]$. As a result,

$$\rho^*[\sigma^*(t)] = \rho[\lambda_2, \sigma(t, \lambda_1)], \forall [t, (\lambda_1, \lambda_2)] \in T \times \Lambda,$$

i.e., (σ, ρ) and (σ^*, ρ^*) are outcome-equivalent. Finally, we show incentive compatibility for both players.

Incentive compatibility for the sender. Suppose the true payoff state is t. Under (σ^*, ρ^*) in the game without language barriers, sending $\sigma^*(t)$ is optimal for the sender. Under (σ, ρ) in the $\mathcal{N}$-dimensional communication game, the equilibrium message of the sender is interpreted by the receiver as $\sigma^*(t)$. If the sender deviates to any other message $m \in M^{\mathcal{N}}$, by (14), it will be interpreted as $\sigma^*(t')$ for some t' and this is (weakly) worse than $\sigma^*(t)$ for the sender under state t.

Incentive compatibility for the receiver. The receiver in an equilibrium (σ^*, ρ^*) of the game without language barriers forms a posterior belief on t upon receiving the messages $\sigma^*(t)$ and chooses the optimal strategy $\rho^*[\sigma^*(t)]$. Note that the receiver in equilibrium (σ, ρ) of the $\mathcal{N}$-dimensional game with language barriers receives two pieces of information, truthfully reported by the sender in the equilibrium, i.e., λ_1 and $\sigma^*(t)$. Since t and λ are independent by Assumption 1, the receiver forms the same posterior belief on t as that under (σ^*, ρ^*). Hence, the same strategy $\rho[\lambda_2, \sigma(t, \lambda_1)] = \rho^*[\sigma^*(t)]$ is a best reply for the receiver. ■

4 Main Results: 1-dimensional Communication

In this section, we focus on 1-dimensional communication to study whether *there exist* language barriers that allow us to do "better" than what we can achieve without them.

In particular, we follow the Goltsman, Hörner, Pavlov, and Squintani (2009) strategy of studying several *modified* versions of cheap-talk communication games, although the games studied here generalize theirs over two dimensions: we consider any arbitrary distribution and any utility functions, while Goltsman, Hörner, Pavlov, and Squintani (2009) focus on the continuous uniform distribution and the quadratic utility function.[10] In Section 4.1, we define mediation equilibria (Goltsman, Hörner, Pavlov, and Squintani (2009)), noisy-talk equilibria (Blume, Board, and Kawamura (2007)), and language-barrier equilibria, all of which may Pareto dominate cheap-talk equilibria. In Section 4.2, we provide a linear ranking regarding the maximal welfare induced by these equilibria.

4.1 Cheap talk communication devices

Recall that a communication game is defined by a tuple $\left\langle M, T, \Lambda, \pi, A, (u_i)_{i\in I}, N\right\rangle$. From now on, we fix the primitives (excluding language barriers), $\left\langle M, T, \pi_T, A, (u_i)_{i\in I}, N=1\right\rangle$, so as to make comparisons meaningful. We define three communication devices.

Mediation equilibria. First, we define mediation equilibria.[11]

Definition 4 $[p : T \longrightarrow \triangle(A)]$ *is a mediation equilibrium if*

$$\int\limits_{a\in A} u_1[t,a]\, p(t)(da) \geq \int\limits_{a\in A} u_1[t,a]\, p(t')(da),\ \forall t,t' \in T, \text{ and} \tag{15}$$

$$\int\limits_{T}\left[\int\limits_{a\in A} u_2[t,a]\, p(t)(da)\right]\pi_T[dt] \geq \int\limits_{T}\left[\int\limits_{a\in A} u_2[t,\iota(a)]\, p(t)(da)\right]\pi_T[dt],\ \forall \iota : A \longrightarrow A. \tag{16}$$

[10] Blume and Board (2010) also undertook an exercise similar to ours. However, they focused on the Goltsman, Hörner, Pavlov, and Squintani (2009) class of games so that our results, which consider more general settings, differ.

[11] In Goltsman, Hörner, Pavlov, and Squintani (2009), $[p : T \longrightarrow \triangle(A)]$ is called an arbitration equilibrium if and only if condition (15) holds. Clearly, a mediation equilibrium is also an arbitration equilibrium, and hence, the maximal utility of arbitration equilibria (weakly) dominates that of mediation equilibria. However, a general welfare comparison between arbitration equilibria and language-barrier equilibria is not available (see Giovannoni and Xiong (2018) for a detailed discussion).

Suppose there is a non-strategic mediator besides the players, who follows $[p : T \longrightarrow \triangle (A)]$ to make recommendations. The sender reports his private payoff state t to the mediator; upon receiving t, the latter commits to drawing from a lottery on A following the distribution $p(t)$; given every realized value a of the lottery, the receiver plays a. A mediation equilibrium requires incentive compatibility of reporting t and playing a by the sender and the receiver, respectively, which are summarized in (15) and (16).

Noisy-talk equilibria. In a noisy-talk game, on top of the primitives, we have a tuple $(\varepsilon, \xi) \in [0,1] \times \triangle (M)$, which has the interpretation that with probability ε, the sender's message is replaced by a random message which is the realization of an exogenous and independent distribution ξ. A potential candidate for a noisy-talk equilibrium is a strategy profile

$$([s : T \longrightarrow \triangle (M)],\ [r : M \longrightarrow \triangle (A)]).$$

Given $[(\varepsilon, \xi), s, r]$, type t of the sender follows $s(t) \in \triangle (M)$ to send a random message; for any realized message m from the sender, with probability $(1-\varepsilon)$, the receiver observes m, and with probability ε, the receiver observes a random message generated by the distribution ξ; finally, upon receiving a (possibly distorted) message m', the receiver takes a random action $r(m') \in \triangle (A)$. We aggregate this process as follows.

$$p^{[(\varepsilon,\xi),\, s,\, r]} : T \longrightarrow \triangle (A), \tag{17}$$

$$p^{[(\varepsilon,\xi),\, s,\, r]}(t)[E] = \int_M \left[(1-\varepsilon) \times r(m)[E] + \varepsilon \times \int_M r(\widetilde{m})[E]\, \xi [d\widetilde{m}] \right] s(t)[dm], \forall E \subset A,$$

i.e., $p^{[(\varepsilon,\xi),\, s,\, r]}(t)$ is the *ex-post* action distribution induced by (s, r), given t. We now define noisy-talk equilibria.

Definition 5 $([s : T \longrightarrow \triangle (M)],\ [r : M \longrightarrow \triangle (A)])$ *is a noisy-talk equilibrium if there exists* $(\varepsilon, \xi) \in [0,1] \times \triangle (M)$ *such that*

$$\forall t \in T, \forall s' : T \longrightarrow \triangle (M), \tag{18}$$

$$\int_{a \in A} u_1(t,a)\, p^{[(\varepsilon,\xi),\, s,\, r]}(t)(da) \geq \int_{a \in A} u_1(t,a)\, p^{[(\varepsilon,\xi),\, s',\, r]}(t)(da),$$

$$\textit{and } \forall r' : M \longrightarrow \triangle(A), \tag{19}$$
$$\int_T \left[\int_{a \in A} u_2(t,a)\, p^{[(\varepsilon,\xi),\, s,\, r]}(t)(da) \right] \pi_T[dt] \geq \int_T \left[\int_{a \in A} u_2(t,a)\, p^{[(\varepsilon,\xi),\, s,\, r']}(t)(da) \right] \pi_T[dt].$$

(18) and (19) in Definition 5 describe the incentive compatibility conditions for the sender and the receiver, respectively. It is worth noting that our notion of noisy talk is the one utilized in Goltsman, Hörner, Pavlov, and Squintani (2009) where the probability of error ε is fixed and independent of messages. Blume, Board, and Kawamura (2007) also consider an alternative notion of noisy talk in which error probabilities are not fixed but are correlated with messages. Our welfare ranking results (i.e., Theorem 2 and Lemma 4) hold for the fixed-noise version of noisy talk, but not the correlated-noise version.[12]

Language-barrier equilibria. A valid language-barrier game is defined by a tuple $[\Lambda,\ \pi \in \triangle(T \times \Lambda)]$ such that the marginal distribution of π on T matches the fixed π_T and assumptions (4) and (5) are satisfied. In game $[\Lambda,\ \pi]$, a potential candidate for a language-barrier equilibrium is a strategy profile

$$[\sigma : T \times \Lambda_1 \rightarrow \triangle(M),\ \rho : \Lambda_2 \times M \rightarrow \triangle(A)].$$

We say $[\sigma,\ \rho]$ is a valid strategy profile if and only if σ and ρ are regular with respect to Λ_1 and Λ_2, respectively, where regularity is as defined in Section 2. Given $(t, \lambda_1, \lambda_2)$, the sender follows $\sigma(t, \lambda_1) \in \triangle(M)$ to send a random message; upon receiving a realized message m, the receiver follows $\rho(\lambda_2, m) \in \triangle(A)$ to play a random action. We use the function $p^{(\sigma,\rho)}$ defined below to aggregate this process.

$$\begin{aligned} p^{(\sigma,\rho)} &: \quad T \times \Lambda_1 \times \Lambda_2 \rightarrow \triangle(A), \\ p^{(\sigma,\rho)}(t, \lambda_1, \lambda_2)[E] &= \int_M [\rho(\lambda_2, m)[E]]\, \sigma(t, \lambda_1)(dm), \forall E \subset A. \end{aligned} \tag{20}$$

Definition 6 *For any valid language-barrier game $(\Lambda,\ \pi)$, we say a valid strategy profile*

$$[\sigma : T \times \Lambda_1 \rightarrow \triangle(M),\ \rho : \Lambda_2 \times M \rightarrow \triangle(A)],$$

[12] For the correlated-noise version of noisy talk, it is obvious that it is welfare dominated by mediation, but a ranking with independent language barriers remains unavailable. The difficulty is that when noise is independent, it can be regarded as (non-strategic) language types of the sender which are independent of payoff states, but when noise is correlated with messages, the independence between noise and payoff states fails.

is a language-barrier equilibrium if

$$\forall\left(t,\lambda_1\right)\in T\times\Lambda_1,\forall\sigma':T\times\Lambda_1\longrightarrow\triangle\left(M\right),\tag{21}$$

$$\int\limits_{\Lambda_2}\left(\int\limits_{a\in A}u_1\left(t,a\right)p^{\left(\sigma,\rho\right)}\left(t,\lambda_1,\lambda_2\right)\left[da\right]-\int\limits_{a\in A}u_1\left(t,a\right)p^{\left(\sigma',\rho\right)}\left(t,\lambda_1,\lambda_2\right)\left[da\right]\right)\pi\left[d\lambda_2\mid t,\lambda_1\right]\geq 0,$$

$$\textit{and }\forall\lambda_2\in\Lambda_2,\forall\rho':\Lambda_2\times M\to\triangle\left(A\right),\tag{22}$$

$$\int\limits_{T\times\Lambda_1}\left(\int\limits_{a\in A}u_2\left(t,a\right)p^{\left(\sigma,\rho\right)}\left(t,\lambda_1,\lambda_2\right)\left[da\right]-\int\limits_{a\in A}u_2\left(t,a\right)p^{\left(\sigma,\rho'\right)}\left(t,\lambda_1,\lambda_2\right)\left[da\right]\right)\pi\left[\left(dt,d\lambda_1\right)\mid\lambda_2\right]\geq 0.$$

Furthermore, we say it is an independent-language-barrier equilibrium, if $t\in T$ and $\lambda\in\Lambda$ are independently distributed according to π.

4.2 Welfare comparison

In this section, we assume $M=\mathbb{R}$ and compare the welfare induced by different notions of equilibrium. As mentioned before, Goltsman, Hörner, Pavlov, and Squintani (2009) and Blume and Board (2010) consider the canonical Crawford and Sobel (1982) setting with quadratic utility, where, in any mediation equilibrium, the sender's expected utility differs from the receiver's expected utility by a constant determined by the "bias." In that setting, it is without loss of generality to compare only the sender's (or the receiver's) expected utility in different equilibria. However, in the general communication model we study here, this simple property no longer holds. We thus introduce a generic social welfare function $\Phi:\mathbb{R}^I\longrightarrow\mathbb{R}$ to aggregate players' utility. That is, if every player $i\in I$ gets expected utility x_i in a given equilibrium, we say this equilibrium achieves social welfare of $\Phi\left[\left(x_i\right)_{i\in I}\right]$. Then, for a fixed social welfare function Φ, let Φ^M, Φ^N, Φ^{LB}, Φ^{ILB} denote the supremum of the social welfare achieved by equilibria in each of our possible protocols (i.e., mediation, noisy talk, language barriers, and independent language barriers, respectively). We now present the main result of this section.

Theorem 2 *For any social welfare function Φ, we have*

$$\Phi^{LB}\geq\Phi^M\geq\Phi^{ILB}\geq\Phi^N.$$

The idea of the proof of Theorem 2 is to show that equilibria with language barriers, mediation, independent language barriers and noisy talk correspond to a series of increasingly restrictive incentive compatibility conditions in that order. The proof is available in Appendix A.3. In Giovannoni and Xiong (2018), we provide an example of $\Phi^{LB} > \Phi^{M}$ and another in which $\Phi^{ILB} > \Phi^{N}$.[13] The latter implies that the $\Phi^{M} = \Phi^{ILB} = \Phi^{N}$ ranking in Goltsman, Hörner, Pavlov, and Squintani (2009) and Blume and Board (2010) does not generalize.

5 Literature Review

The literature on strategic communication is very large but in almost all of this literature, the assumption is that language ability is never an issue.[14] A significant exception is Farrell (1993), where the issue of how exactly information is transmitted is taken seriously but there is a "rich language assumption", which excludes language barriers, and the crucial restriction is that messages come with some intrinsic meaning. Thus, for Farrell (1993), the restriction is not that players cannot use or understand some messages but rather that, whenever credible, messages should be taken literally.

Still, a few authors have argued that language is necessarily too coarse for communication in certain environments. For example, Arrow (1975) discusses the reasons of organizational codes and both Crémer, Garicano, and Prat (2007) and Sobel (2015) model such codes by using a setting where messages are too few to avoid ambiguity. While

[13] In the latter example, we construct an independent-language-barrier equilibrium which achieves almost full revelation. To achieve the same utility with noisy-talk, there should be enough noise to soften the conflict of interest between the sender and the receiver. However, noise does not carry any information regarding payoff states, and hence, it brings significant deadweight losses in welfare, because upon receiving noise, the receiver must take an action without information regarding payoff states. These two forces go in opposite directions, and in this example, the latter dominates the former, so that $\Phi^{ILB} > \Phi^{N}$. Whether $\Phi^{M} > \Phi^{ILB}$ in some environments or $\Phi^{M} = \Phi^{ILB}$ for all environments remains an open question.

[14] Part of this literature introduces frictions in communication, but these are never interpreted as language barriers. For example, beginning with Milgrom (1981), there is significant amount of work that considers communication when messages are (possibly costless) evidence so that lying is not allowed. Similarly, in Kartik (2009) lying is costly. Chen (2011) and Kartik, Ottaviani, and Squintani (2007) consider the case of naive receivers, so that they face cognitive limitations, rather than language barriers.

our results suggest that multi-dimensional communication can overcome all such issues, there may be reasons in those environments, such as complexity or the time needed to develop and understand such messages, that pose substantial limits on how much can be done with them. The closest work to ours is Blume and Board (2013) which introduce the notion of language types and use it to describe language barriers. We address different issues, however, as Blume and Board (2013) focus on indeterminacy of meaning in communication.[15] They consider common-interest games and show that, for any fixed game without language barriers, the most efficient equilibrium of a similar game with language-type space Λ (and 1-dimensional communication) displays indeterminacy of meaning, if Λ satisfies *a full-support assumption*. We, on the other hand, study the welfare effects of language barriers in strategic communication. Furthermore, our results do not have direct implications for those of Blume and Board (2013) and vice-versa.[16]

Furthermore, we should note that Blume (2018) extends the Blume and Board (2013) analysis by looking at the issues raised by language barriers in a sender-receiver common-interest context where the sender still has private information about her language type but there is no common prior on it. We do not focus on higher-order uncertainty.[17] As discussed, in our paper we also look at whether particular language barriers can improve upon communication in non-common interest settings. A few papers have particular relevance to our work here. Krishna and Morgan (2004) show that more (Pareto) efficient equilibria may be obtained by allowing for the informed sender and uninformed receiver to exchange messages at a first stage and then allowing the sender to send a second message, i.e. a conversation. The N-dimensional communication in our setting should not be interpreted as a conversation because the one-way communication takes place in a single stage, compared to the two-way and multi-stage communication in a conversation.

[15] Indeterminacies of meaning arise when, in the presence of language barriers, players's equilibrium strategies are such that they would want to deviate if they knew their opponent's language type.

[16] For a common-interest game, Corollary 1 in Appendix B.1 states that given any language barriers, we can (arbitrarily) approximate full efficiency with N-dimensional communication for large enough N. Full efficiency would guarantee determinacy of meaning, but for any given N only approximate efficiency is achieved and so indeterminacies cannot be ruled out. On the other hand, such indeterminacies are not guaranteed to exist either, because our regularity conditions on how language barriers (i.e., Λ) are extended to N-dimensional communication imply that the full support assumption (a sufficient, but not necessary, condition) in Blume and Board (2013) will be violated.

[17] However, Theorem 1 holds for language-ex-post as well as language-interim equilibria so it would still hold in situations of higher order uncertainty.

Blume, Board, and Kawamura (2007) show that the exogenously given possibility of an error in communication actually improves communication in equilibrium, while in our setting it is exogenous language barriers that provide such results. In fact, Goltsman, Hörner, Pavlov, and Squintani (2009) provide an upper bound on *ex-ante* welfare if mediation is introduced in the model and show that conversations (sometimes) and noisy talk (always) can reach, but not surpass such a bound.[18] Blume and Board (2010) study language barriers under the assumption of independence between language types and payoff states and argue that the welfare bound can be reached by language barriers. We extend these results to a class of much more general communication games and provide a linear ranking amongst all of these communication protocols. In particular, we show that under the independence assumption, but in this general setting, the optimal language barriers will always do no worse than the optimal noisy talk. Indeed, in Giovannoni and Xiong (2018) we provide an example where the optimal independent-language-barrier equilibrium does strictly better than the optimal noisy-talk equilibrium, which implies that, in general and in contrast with the conclusions drawn in Goltsman, Hörner, Pavlov, and Squintani (2009) and Blume and Board (2010), noisy communication cannot always achieve the welfare bound obtained through mediated communication. We also go beyond the independence assumption between payoff states and language types and show that the optimal language barriers can do better than mediation, whereas a comparison with arbitration cannot be made without specifying the form of arbitration or the welfare function.[19] Finally, Blume, Lai, and Lim (2017) find some laboratory evidence that randomized responses increase information transmission.

Some of our communication protocols in this latter part of the paper correspond to some of the equilibrium notions developed in the literature on correlated equilibria in games of incomplete information. It is easy to check, for example, that mediation corresponds to a communication equilibrium in Forges (1993). For language barriers, however, things are not so simple. At an intuitive level, it is clear that language barriers bring correlation to the communication between the sender and the receiver, but it is *not obvious* how the restrictions imposed by language barriers (with or without independence) could

[18] Ganguly and Ray (2011) argue that any noisy communication protocol requires a larger set of messages than those used in the standard Crawford and Sobel (1982) setting. They show that simple mediation, where no more messages can be used than in the corresponding Crawford and Sobel (1982) setting, does not improve on such setting.

[19] See Giovannoni and Xiong (2018) for details.

be translated to any notion of correlated equilibrium.[20] This implies that all of the results obtained here cannot be established by mapping our equilibrium notions to those in the correlated equilibrium literature. Thus, our results can be seen as providing a (further) link between the literature on cheap talk with different forms of commitment or noise and the literature of correlation in games of incomplete information, but the full implication of such links is still awaiting a more systematic analysis.

6 Conclusion

We conclude the paper by discussing some features of our model and some potential alternative ways to model language barriers. In our N-dimensional communication, we start with primitives defined in 1-dimensional messages, and then extend to N-dimensional messages. One point worth emphasizing is that in our protocol, the sender communicates an N-dimensional message in one shot, so that the protocol cannot be interpreted as "a conversation". Still, one could interpret the protocol as one with 1-dimensional communication but where the set of messages M has a structure where each available message is a (N-dimensional) vector, with each 1-dimensional component belonging to a common set of basic components, like an alphabet. In such interpretation, our restrictions would be that any two messages which have the same basic components must belong to the same language type. The challenge with such interpretation is that of explaining why these restrictions are the natural ones and this is beyond the scope of this paper.[21]

On the other hand, when we extend the restrictions implied by language barriers on M to language barriers on M^N, we do so in a "natural" way in the sense that such extensions take the original language barriers literally. So, a sender who only understands messages in λ_1 is only able to send messages only in $(\lambda_1)^N$ while a receiver is only able

[20] Language barriers introduce exogenous constraints on what messages the sender can send and the receiver can understand. In particular, there may exist a language-barrier equilibrium in which such barriers forbid a sender to deviate to a particular message which is more profitable, and similarly, such barriers forbid a receiver to deviate to a better strategy that is not measurable for her language type. Such profitable deviations are allowed in correlated equilibria, and as a result, language-barrier equilibria cannot be translated to a notion of correlated equilibrium.

[21] Also, how should we interpret a dimension in a message, if a message is taken as a minimal unit in communication?

to distinguish $x, y \in M^N$ if $x \nsim_{\lambda_2} y$. Clearly, these are strong restrictions but one could assume weaker restrictions and our results would still hold.[22]

Finally, we may ask whether there are other, simpler, protocols which might be equally effective as N-dimensional communication. For instance, it would seem too strong to require all messages to be N-dimensional, instead of allowing for messages of any dimensional size, up to N, because the latter should be simpler and, maybe, equally effective: players may use the length of a message to transmit information. In fact, that such protocol would be an improvement over ours is not so obvious. First of all, note that if we try to model real-life communication, then resources (e.g. time, effort) will be limited and this implies that, necessarily, the number of messages that can be transmitted must be *finite* and *bounded* and the "N" in our protocol models this upper bound. Given this, a protocol that allows for varied dimensional sizes can always be transformed into our protocol by adding to the latter a message (e.g. "silence") that is available to all language types of the sender. Thus, it is without loss of generality to consider N-dimensional messages with a fixed N. Furthermore, given the upper bound "N", only finite messages can be transmitted by counting the dimensions of a message. As a result, infinite-message equilibria would not be effectively replicated under the modified protocol, i.e., Theorem 3 in Appendix B.2 would not hold.

But even for finite-message equilibria, counting message length encounters a practical difficulty: a language type can easily count the length of a message if he fully understands the message, but he can hardly do it otherwise. Therefore, in presence of language barriers, counting message length is not an effective scheme. In our proof of Theorems 1 and 3, we (implicitly) assume receivers have the ability to count but such an assumption is for notational simplicity only, and can be dropped. It can be easily shown that it is always possible to design messages where each receiver type is assigned a unique opening and/or ending portion (which such type understands) so that any such type can scan the message without counting and still know where the message addressed to him is.

[22]For example, suppose $\{*, \#\} \cap \lambda = \varnothing$. In our model, when $N = 2$, the receiver cannot distinguish between messages $(*, *), (*, \#), (\#, *)$ and $(\#, \#)$. But, as pointed out by a refeee, it is conceivable that a receiver that does not understand the difference between $*$ and $\#$ may still be able to recognize repeating patterns. Thus, even though he cannot distinguish $(*, *)$ from $(\#, \#)$, and $(*, \#)$ from $(\#, *)$, he may be able to distinguish any message in $\{(*, *), (\#, \#)\}$ from any message in $\{(*, \#), (\#, *)\}$. Clearly, in this latter case, the extension to multi-dimensional language barriers is less restrictive than the one we assume in the paper.

A Additional Proofs

A.1 Proof of Lemma 1

Recall $\overline{N} > 3 + |\Lambda_1|$. Label the elements in Λ_1 as $\lambda_1^{(1)}$, $\lambda_1^{(2)}$, ..., $\lambda_1^{(K)}$, where $K = |\Lambda_1| < \overline{N}$.

For each $\lambda_1^{(k)} \in \Lambda_1$ with $k \leq K$, we have $\lambda_1^{(k)} \cap \lambda_2 \neq \varnothing$ and $\left|\lambda_1^{(k)}\right| \geq 2$, due to (4) and (5). Thus, we fix some

$$m^{(k)} \in \lambda_1^{(k)} \cap \lambda_2, \tag{23}$$

and some $\widetilde{m}^{(k)} \in \lambda_1^{(k)} \diagdown \left\{m^{(k)}\right\}$, i.e., $m^{(k)} \neq \widetilde{m}^{(k)}$. Note that

$$m^{(k)} \nsim_{\lambda_2} \widetilde{m}^{(k)}, \tag{24}$$

whether $\widetilde{m}^{(k)} \in \lambda_2$ or $\widetilde{m}^{(k)} \notin \lambda_2$.

Then, define $\Upsilon_{\lambda_2} : \Lambda_1 \longrightarrow M^{\overline{N}}$ as follows. For each $k \in \{1, 2, ..., K\}$,

$$\Upsilon_{\lambda_2}\left[\lambda_1^{(k)}\right] = [m_l]_{l=1}^{\overline{N}} \in M^{\overline{N}} \text{ such that } m_l = \begin{cases} m^{(k)}, & \text{if } l = k; \\ \widetilde{m}^{(k)}, & \text{otherwise.} \end{cases}$$

That is, type $\lambda_1^{(k)}$ uses $m^{(k)}$ to denote "yes" and $\widetilde{m}^{(k)}$ for "no". Furthermore, the sender associates each of the first K dimensions of the message $\Upsilon_{\lambda_2}\left[\lambda_1^{(k)}\right]$ to one element in Λ_1, and reveals whether her language type is that element in the associated dimension. Precisely, $\lambda_1^{(k)}$ says "yes" (i.e., $m^{(k)}$) in the k-th dimension, and "no" (i.e., $\widetilde{m}^{(k)}$) in all other dimensions.

For $k \neq k'$, we show $\Upsilon_{\lambda_2}\left[\lambda_1^{(k)}\right] \nsim_{\lambda_2} \Upsilon_{\lambda_2}\left[\lambda_1^{(k')}\right]$, as needed in (7). By the definition of Υ_{λ_2}, we have

$$\begin{aligned} \Upsilon_{\lambda_2}\left[\lambda_1^{(k)}\right] &= [m_l]_{l=1}^{\overline{N}} = \left[m_k = m^{(k)}, \ \left(m_l = \widetilde{m}^{(k)}\right)_{l \neq k}\right]; \\ \Upsilon_{\lambda_2}\left[\lambda_1^{(k')}\right] &= [\widehat{m}_l]_{l=1}^{\overline{N}} = \left[\widehat{m}_{k'} = m^{(k')}, \ \left(\widehat{m}_l = \widetilde{m}^{(k')}\right)_{l \neq k'}\right]. \end{aligned}$$

Consider two cases: (1) $m^{(k)} \neq \widetilde{m}^{(k')}$ and (2) $m^{(k)} = \widetilde{m}^{(k')}$. In case (1), $m^{(k)} \neq \widetilde{m}^{(k')}$ and $m^{(k)} \in \lambda_2$ (by (23)) imply

$$m_k = m^{(k)} \nsim_{\lambda_2} \widetilde{m}^{(k')} = \widehat{m}_k,$$

i.e., in the k-th dimension, $m_k \not\sim_{\lambda_2} \widehat{m}_k$, which further implies $\Upsilon_{\lambda_2}\left[\lambda_1^{(k)}\right] \not\sim_{\lambda_2} \Upsilon_{\lambda_2}\left[\lambda_1^{(k')}\right]$.

In case (2), recall $\overline{N} > 3 + |\Lambda_1| > 3$ by (6). Pick any $k'' \in \{1, ..., \overline{N}\} \diagdown \{k, k'\}$. Then, (24) implies

$$m_{k''} = \widetilde{m}^{(k)} \not\sim_{\lambda_2} m^{(k)} = \widetilde{m}^{(k')} = \widehat{m}_{k''},$$

i.e., in the k''-th dimension, $m_{k''} \not\sim_{\lambda_2} \widehat{m}_{k''}$, which further implies $\Upsilon_{\lambda_2}\left[\lambda_1^{(k)}\right] \not\sim_{\lambda_2} \Upsilon_{\lambda_2}\left[\lambda_1^{(k')}\right]$
■

A.2 Proof of Lemma 2

Fix any $(\lambda_1, \lambda_2) \in \Lambda$. Recall $\lambda_1 \cap \lambda_2 \neq \varnothing$ and $|\lambda_1| \geq 2$. Thus, we fix some $m \in \lambda_1 \cap \lambda_2$, and some $\widetilde{m} \in \lambda_1 \diagdown \{m\}$, i.e., $m \neq \widetilde{m}$. Note that

$$m \not\sim_{\lambda_2} \widetilde{m}, \tag{25}$$

whether $\widetilde{m} \in \lambda_2$ or $\widetilde{m} \notin \lambda_2$.

Recall $\widehat{N} > \left|\mathcal{E}^{(\sigma^*, \rho^*)}\right|$ by (8). Label the elements in $\mathcal{E}^{(\sigma^*, \rho^*)}$ as $m^{(1)}$, $m^{(2)}$, ..., $m^{(K)}$, where $K = \left|\mathcal{E}^{(\sigma^*, \rho^*)}\right| < \widehat{N}$. Then, define $\Gamma_{(\lambda_1, \lambda_2)} : \mathcal{E}^{(\sigma^*, \rho^*)} \longrightarrow (\lambda_1)^{\widehat{N}}$ as follows. For each $k \in \{1, 2, ..., K\}$,

$$\Gamma_{(\lambda_1, \lambda_2)}\left[m^{(k)}\right] = [m_l]_{l=1}^{\widehat{N}} \in M^{\widehat{N}} \text{ such that } m_l = \begin{cases} m, & \text{if } l = k; \\ \widetilde{m}, & \text{otherwise.} \end{cases}$$

That is, type λ_1 use m to denote "yes" and $\widetilde{m}$ for "no". Furthermore, λ_1 associates each of the first K dimensions of the message $\Gamma_{(\lambda_1, \lambda_2)}\left[m^{(k)}\right]$ to one element in $\mathcal{E}^{(\sigma^*, \rho^*)}$, and reveals whether she intends to send that element in the associated dimension. Precisely, to send the message $m^{(k)} \in \mathcal{E}^{(\sigma^*, \rho^*)}$, λ_i says "yes" (i.e., m) in the k-th dimension, and "no" (i.e., $\widetilde{m}$) in all other dimensions.

For $k \neq k'$, we show $\Gamma_{(\lambda_1, \lambda_2)}\left[m^{(k)}\right] \not\sim_{\lambda_2} \Gamma_{(\lambda_1, \lambda_2)}\left[m^{(k')}\right]$, as needed in (9). By the definition of $\Gamma_{(\lambda_1, \lambda_2)}$, we have

$$\Gamma_{(\lambda_1, \lambda_2)}\left[m^{(k)}\right] = [m_l]_{l=1}^{\widehat{N}} = \left[m_k = m, \ (m_l = \widetilde{m})_{l \neq k}\right];$$
$$\Gamma_{(\lambda_1, \lambda_2)}\left[m^{(k')}\right] = [\widehat{m}_l]_{l=1}^{\widehat{N}} = \left[\widehat{m}_{k'} = m, \ (\widehat{m}_l = \widetilde{m})_{l \neq k'}\right].$$

Since $k \neq k'$, (25) implies

$$m_k = m \not\sim_{\lambda_2} \widetilde{m} = \widehat{m}_k,$$

i.e. in the k-th dimension, $m_k \not\sim_{\lambda_2} \widehat{m}_k$, which further implies $\Gamma_{(\lambda_1,\lambda_2)}\left[m^{(k)}\right] \not\sim_{\lambda_2} \Gamma_{(\lambda_1,\lambda_2)}\left[m^{(k')}\right]$
■

A.3 Proof of Theorem 2

We first introduce the notion of weak language-barrier equilibrium, which differs from language-barrier equilibria (resp. independent-language-barrier equilibria) because in the former we remove the assumption

$$|\lambda_i| \geq 2, \forall (i, \lambda) \in I \times \Lambda. \tag{26}$$

That is, every language type must be endowed with at least two messages in any language-barrier equilibrium, but language types in a weak-language-barrier equilibrium may be endowed with just one message. Clearly, a language-barrier equilibrium is a weak language-barrier equilibrium. Conversely, Lemma 3 below establishes that for any weak language-barrier equilibrium, there is an outcome-equivalent language-barrier equilibrium. Because of this, it is without loss of generality to focus on weak language-barrier equilibria. Given this, the proof of Theorem 2 follows immediately from Lemmas 4, 5 and 6.

Lemma 3 *For any weak language-barrier equilibrium, there exists an outcome-equivalent language-barrier equilibrium. Furthermore, for any weak independent-language-barrier equilibrium, there exists an outcome-equivalent independent-language-barrier equilibrium.*

Proof Fix any valid language-barrier game (Λ, π), and any weak language-barrier equilibrium,

$$[\sigma : T \times \Lambda_1 \to \triangle(M), \ \rho : \Lambda_2 \times M \to \triangle(A)].$$

Recall $M = \mathbb{R}$. Pick any disjoint $M^* (\subset M)$ and $M^{**} (\subset M)$ which are both homeomorphic to M, e.g., $M^* = \left(0, \frac{1}{3}\right)$ and $M^* = \left(\frac{2}{3}, 1\right)$. Let

$$\gamma^* : M \longrightarrow M^*,$$
$$\gamma^{**} : M \longrightarrow M^{**},$$

denote the homeomorphisms, and let γ^{*-1} and γ^{**-1} denote the inverse functions.

Define a new valid language-barrier game $\left(\widetilde{\Lambda},\ \widetilde{\pi}\right)$ where

$$\begin{aligned}
\widetilde{\Lambda}_1 &= \{\gamma^*(\lambda_1) \cup \gamma^{**}(\lambda_1) : \lambda_1 \in \Lambda_1\};\\
\widetilde{\Lambda}_2 &= \{\gamma^*(\lambda_2) \cup \gamma^{**}(\lambda_2) : \lambda_2 \in \Lambda_2\};
\end{aligned}$$
$$\widetilde{\pi}(E) = \pi\left(\{[t, \lambda_1, \lambda_2] : [t,\ \gamma^*(\lambda_1) \cup \gamma^{**}(\lambda_1),\ \gamma^*(\lambda_2) \cup \gamma^{**}(\lambda_2)] \in E\}\right), \forall E \subset T \times 2^M \times 2^M,$$

i.e., any sender language type λ_1 is transformed into a new type $\gamma^*(\lambda_1) \cup \gamma^{**}(\lambda_1)$ which contains two copies of the original type, with the first copy transformed from λ_1 via γ^* and the second copy from λ_1 via γ^{**}; a similar construction applies to the receiver's language types; the new prior $\widetilde{\pi}$ inherits the distribution from the original prior π.

For any $\mu \in \triangle(M)$, define $\gamma^*(\mu) \in \triangle(M^*)$ as

$$\gamma^*(\mu)[E] = \mu\left(\gamma^{*-1}[E]\right), \forall E \subset M^*,$$

i.e., for any random message generated by μ, we transform it to a message in M^* via γ^*, and $\gamma^*(\mu)$ is the distribution of the transformed message from μ.

For each $\lambda_2 \in \Lambda_2$ such that $\lambda_2 \subsetneqq M$, fix any $m^{\lambda_2} \in M \diagdown \lambda_2$. Furthermore, if $\lambda_2 = M$, fix any $m^{\lambda_2} \in M$. The sole purpose of construction of m^{λ_2} is for the measurability (with respect to λ_2) of $\widetilde{\rho}$ defined below. We now define the outcome-equivalent language-barrier equilibrium.

$$\left[\widetilde{\sigma} : T \times \widetilde{\Lambda}_1 \to \triangle(M),\ \widetilde{\rho} : \widetilde{\Lambda}_2 \times M \to \triangle(A)\right],$$
$$\widetilde{\sigma}[t,\ \gamma^*(\lambda_1) \cup \gamma^{**}(\lambda_1)] = \gamma^*(\sigma[t,\ \lambda_1]),$$
$$\widetilde{\rho}[\gamma^*(\lambda_2) \cup \gamma^{**}(\lambda_2),\ m] = \begin{cases} \rho\left[\lambda_2,\ \gamma^{*-1}(m)\right], & \text{if } m \in M^*;\\ \rho\left[\lambda_2,\ \gamma^{**-1}(m)\right], & \text{if } m \in M^{**};\\ \rho\left[\lambda_2,\ m^{\lambda_2}\right], & \text{otherwise.} \end{cases}$$

That is, a new sender's type $\gamma^*(\lambda_1) \cup \gamma^{**}(\lambda_1)$ follows the strategy $\sigma[t,\ \lambda_1]$ of the old type λ_1, but transforms the (random) message into a message in M^* via γ^*; a new receiver's type $\gamma^*(\lambda_2) \cup \gamma^{**}(\lambda_2)$ first decodes the messages in M^* and M^{**} via γ^{*-1} and γ^{**-1}, respectively, and then follows the strategies $\rho\left[\lambda_2,\ \gamma^{*-1}(m)\right]$ and $\rho\left[\lambda_2,\ \gamma^{**-1}(m)\right]$ of the old type λ_2.

By the definition of $\widetilde{\Lambda}_1$ and $\widetilde{\Lambda}_2$, the sender sends messages only in M^* and M^{**}, because $\gamma^*(M) = M^*$ and $\gamma^{**}(M) = M^{**}$. Second, the receiver treats M^* and M^{**} as the transformed copies of the same set M (via γ^* and γ^{**}, respectively). Hence, it is without loss of generality for the sender to send messages only in M^*.[23] Given this, $[\widetilde{\sigma},\ \widetilde{\rho}]$ just replicates $[\sigma,\ \rho]$, and $[\widetilde{\sigma},\ \widetilde{\rho}]$ inherits the incentive compatibility properties of $[\sigma,\ \rho]$. Therefore, $[\widetilde{\sigma},\ \widetilde{\rho}]$ is an outcome-equivalent language-barrier equilibrium. A similar argument applies to weak independent-language-barrier equilibria ∎

Lemma 4 *For any noisy talk equilibrium, there exists an outcome-equivalent independent-language-barrier equilibrium.*

Proof In light of Lemma 3, it is without loss of generality for us to focus on weak independent-language-barrier equilibria. Fix any noisy-talk game $(\varepsilon, \xi) \in [0,1] \times \triangle(M)$, and any noisy-talk equilibrium $([s : T \longrightarrow \triangle(M)],\ [r : M \longrightarrow \triangle(A)])$ in the game. Define a language-barrier game $(\Lambda,\ \pi)$, such that the random vectors defined on T and Λ are independent under π, and

$$\lambda^M \equiv M \text{ and } \lambda^m \equiv \{m\}, \forall m \in M;$$
$$\Lambda_1 = \left\{\lambda^M\right\} \cup \{\lambda^m : m \in M\};$$
$$\Lambda_2 = \left\{\lambda^M\right\};$$
$$\pi_\Lambda\left[\left\{\lambda^M\right\} \times \left\{\lambda^M\right\}\right] = 1 - \varepsilon;$$
$$\pi_\Lambda\left[E \times \left\{\lambda^M\right\}\right] = \varepsilon \times \xi\left[\{m : \lambda^m \in E\}\right], \forall E \subset 2^M \diagdown \{M\}.$$

That is, the receiver understands all messages in M; with probability $1 - \varepsilon$, the sender understand all messages in M, and with probability ε, the sender is endowed with a single message; conditional on the probability-ε event, the distribution follows ξ, with $\{m\}$ replacing m.

Then, we define a weak independent-language-barrier equilibrium

$$[\sigma : T \times \Lambda_1 \to \triangle(M),\ \rho : \Lambda_2 \times M \to \triangle(A)],$$

[23] Any message in M^{**} has a corresponding message M^* which plays the same role.

such that for every $(t, m) \in T \times M$,

$$\sigma\left(t,\ \lambda_1 = \lambda^M\right) = s(t),$$
$$\sigma(t,\ \lambda_1 = \lambda^m) = \delta_m,$$
$$\rho\left(\lambda^M,\ m\right) = r(m),$$

where δ_m denotes the Dirac measure on m. Clearly, incentive compatibility for every $\lambda^m \equiv \{m\}$ is satisfied. Then, the incentive compatibility of the sender's language type $\lambda^M \equiv M$ and the receiver's language type $\lambda^M \equiv M$ in $[\sigma,\ \rho]$ inherits the incentive compatibility of the sender and the receiver in the noisy-talk equilibrium $(s,\ r)$, respectively. I.e., $[\sigma,\ \rho]$ is an outcome-equivalent weak independent-language-barrier equilibrium. Finally, by Lemma 3, an outcome-equivalent independent-language-barrier equilibrium exists ■

Lemma 5 *For any independent-language-barrier equilibrium, there exists an outcome-equivalent mediation equilibrium.*

Proof Fix any valid language-barrier game $(\Lambda,\ \pi)$, and any independent-language-barrier equilibrium

$$[\sigma : T \times \Lambda_1 \to \triangle(M),\ \ \rho : \Lambda_2 \times M \to \triangle(A)],$$

Recall $p^{(\sigma,\rho)} : T \times \Lambda_1 \times \Lambda_2 \to \triangle(A)$ defined in (20):

$$p^{(\sigma,\rho)}(t, \lambda_1, \lambda_2)[E] = \int_M [\rho(\lambda_2, m)[E]]\, \sigma(t, \lambda_1)(dm),\ \forall E \subset A.$$

i.e., $p^{(\sigma,\rho)}(t, \lambda_1, \lambda_2)$ is the *ex-post* action distribution induced by $[\sigma,\ \rho]$, given $(t, \lambda_1, \lambda_2)$. Then, define

$$\mathcal{P}^{(\sigma,\rho)} : T \to \triangle(A),$$
$$\mathcal{P}^{(\sigma,\rho)}(t)[E] = \int_\Lambda \left[p^{(\sigma,\rho)}(t, \lambda_1, \lambda_2)[E]\right] \pi_\Lambda[d\lambda_1, d\lambda_2],\ \forall E \subset A, \tag{27}$$

i.e., $\mathcal{P}^{(\sigma,\rho)}(t)$ is the *ex-post* action distribution induced by $[\sigma,\ \rho]$, given t. We now show that $\mathcal{P}^{(\sigma,\rho)} : T \to \triangle(A)$ defined above is a mediation equilibrium. First, since $[\sigma,\ \rho]$ is a language-barrier equilibrium, (21) in Definition 6 implies

$$\forall (t, \lambda_1) \in T \times \Lambda_1, \forall \sigma' : T \times \Lambda_1 \longrightarrow \triangle (M),$$
$$\int\limits_{\Lambda_2} \left(\int\limits_{a \in A} u_1 (t, a)\, p^{(\sigma, \rho)} (t, \lambda_1, \lambda_2) [da] - \int\limits_{a \in A} u_1 (t, a)\, p^{(\sigma', \rho)} (t, \lambda_1, \lambda_2) [da] \right) \pi [d\lambda_2 \mid t, \lambda_1] \geq 0, \tag{28}$$

Recall that $t \in T$ and $\lambda \in \Lambda$ are independently distributed under π, and hence (28) reduces to

$$\int\limits_{\Lambda_2} \left(\int\limits_{a \in A} u_1 (t, a)\, p^{(\sigma, \rho)} (t, \lambda_1, \lambda_2) [da] - \int\limits_{a \in A} u_1 (t, a)\, p^{(\sigma', \rho)} (t, \lambda_1, \lambda_2) [da] \right) \pi [d\lambda_2 \mid \lambda_1] \geq 0. \tag{29}$$

Given the definition of $\mathcal{P}^{(\sigma, \rho)}$ in (27), if we integrate (29) over Λ_1, we get

$$\int\limits_{a \in A} u_1 [t, a]\, \mathcal{P}^{(\sigma, \rho)} (t) [da] \geq \int\limits_{a \in A} u_1 [t, a]\, \mathcal{P}^{(\sigma', \rho)} (t) [da], \forall t, \sigma'. \tag{30}$$

Finally, for every $t' \in T$, consider $\sigma' (t, \lambda_1) \equiv \sigma (t', \lambda_1)$, and (30) becomes

$$\int\limits_{a \in A} u_1 [t, a]\, \mathcal{P}^{(\sigma, \rho)} (t) [da] \geq \int\limits_{a \in A} u_1 [t, a]\, \mathcal{P}^{(\sigma, \rho)} \left(t'\right) [da], \forall t, t' \in T,$$

i.e., (15) in Definition 4 holds.

Second, since $[\sigma, \ \rho]$ is a language-barrier equilibrium, (22) in Definition 6 implies

$$\text{and } \forall \lambda_2 \in \Lambda_2, \forall \rho' : \Lambda_2 \times M \rightarrow \triangle (A),$$
$$\int\limits_{T \times \Lambda_1} \left(\int\limits_{a \in A} u_2 (t, a)\, p^{(\sigma, \rho)} (t, \lambda_1, \lambda_2) [da] - \int\limits_{a \in A} u_2 (t, a)\, p^{(\sigma, \rho')} (t, \lambda_1, \lambda_2) [da] \right) \pi [(dt, d\lambda_1) \mid \lambda_2] \geq 0. \tag{31}$$

Given the definition of $\mathcal{P}^{(\sigma, \rho)}$ defined in (27), if we integrate (31) over Λ_2, it becomes

$$\int\limits_{T} \left[\int\limits_{a \in A} u_2 [t, a]\, \mathcal{P}^{(\sigma, \rho)} (t) (da) \right] \pi_T [dt] \geq \int\limits_{T} \left[\int\limits_{a \in A} u_2 (t, a)\, \mathcal{P}^{(\sigma, \rho')} (t) (da) \right] \pi_T [dt], \forall \rho',$$

which further implies

$$\forall \iota : A \longrightarrow A,$$
$$\int\limits_{T} \left[\int\limits_{a \in A} u_2 [t, a]\, \mathcal{P}^{(\sigma, \rho)} (t) (da) \right] \pi_T [dt] \geq \int\limits_{T} \left[\int\limits_{a \in A} u_2 [t, \iota (a)]\, \mathcal{P}^{(\sigma, \rho)} (t) (da) \right] \pi_T [dt],$$

i.e., (16) in Definition 4 holds. Therefore, $\mathcal{P}^{(\sigma,\rho)} : T \to \triangle(A)$ is a mediation equilibrium ■

Lemma 6 *For any mediation equilibrium, there exists an outcome-equivalent language-barrier equilibrium.*

Proof Fix any mediation equilibrium $[p : T \longrightarrow \triangle(A)]$ so that, in particular,

$$\forall \iota : A \longrightarrow A,$$
$$\int_T \left[\int_{a\in A} u_2[t,a]\, p(t)(da) \right] \pi_T[dt] \geq \int_T \left[\int_{a\in A} u_2[t,\iota(a)]\, p(t)(da) \right] \pi_T[dt]. \tag{32}$$

In light of Lemma 3, it is without loss of generality for us to focus on weak language-barrier equilibria. Recall $M = A = \mathbb{R}$. Define a language-barrier game (Λ, π), such that

$$\lambda^M \equiv M \text{ and } \lambda^a \equiv \{a\}, \forall a \in A = M;$$
$$\Lambda_1 = \{\lambda^a : a \in A = M\},$$
$$\Lambda_2 = \left\{\lambda^M\right\},$$
$$\pi[E] = \int_T p(t) \left[\left\{ a : \left(t, \lambda^a, \lambda^M\right) \in E \right\} \right] \pi_T[dt], \forall E \subset T \times 2^M \times 2^M,$$

i.e., the receiver has a unique language type $\lambda^M \equiv M$, who understands all messages; the sender's language types have the form $\lambda^a \equiv \{a\}$ for $a \in A = M$; conditional on payoff state t, the distribution $\pi\left(\lambda^a, \lambda^M \mid t\right)$ inherits the distribution from $p(t)[a]$, with $\lambda^a \equiv \{a\}$ replacing a.

Define $[\sigma : T \times \Lambda_1 \to \triangle(M),\ \rho : \Lambda_2 \times M \to \triangle(A)]$ as follows

$$\sigma[t,\ \lambda_1 = \lambda^a] = \delta_a, \forall a \in A = M,$$
$$\rho\left[\lambda_2 = \lambda^M,\ m = a\right] = \delta_a, \forall a \in A = M,$$

where δ_a is the Dirac measure on a. Clearly, incentive compatibility of each sender's language type $\lambda^a \equiv \{a\}$ is satisfied. The incentive compatibility of the receiver follows from (32). More specifically, $p^{(\sigma,\rho)} : T \times \Lambda_1 \times \Lambda_2 \to \triangle(A)$ as defined in (20) has the value

$$p^{(\sigma,\rho)}\left[t,\ \lambda_1 = \lambda^a,\ \lambda_2 = \lambda^M\right] = \delta_a.$$

And hence, (32) implies

$$\forall \lambda_2 \in \Lambda_2, \forall \rho' : \Lambda_2 \times M \to \triangle(A),$$

$$\int\limits_{T\times\Lambda_1} \left(\int\limits_{a\in A} u_2(t,a)\, p^{(\sigma,\rho)}(t,\lambda_1,\lambda_2)[da] - \int\limits_{a\in A} u_2(t,a)\, p^{(\sigma,\rho')}(t,\lambda_1,\lambda_2)[da] \right) \pi[(dt,d\lambda_1) \mid \lambda_2] \geq 0,$$

i.e., incentive compatibility of the receiver is satisfied, and $[\sigma, \ \rho]$ is an outcome-equivalent weak language-barrier equilibria. Finally, by Lemma 3, an outcome-equivalent independent-language-barrier equilibrium exists. ■

B Additional Material on N-Dimensional Communication

B.1 Implications of Theorem 1

One immediate implication of Theorem 1 is that any language barriers in the canonical Crawford and Sobel (1982) cheap-talk model can be overcome. In that model, there exists a maximally-revealing equilibrium, in which finite messages are transmitted. Hence, all equilibria in the model are finite-message equilibria, and Theorem 1 immediately implies that all of them can be replicated whatever language barriers there are, if multi-dimensional communication is allowed.

A second, less immediate, implication focuses on the setting studied by Blume and Board (2013), which is that of a common-interest sender-receiver game. Specifically, we assume the following:

Assumption 2 (common-interest sender-receiver game) *$u_1 \equiv u_2 \equiv u$ is continuous and T and A are compact metric spaces.*

We use u to denote the common utility function for both players. In this setting, Blume and Board (2013) prove that indeterminacies of meaning are inevitably induced by language barriers under 1-dimensional communication, which implies, as a by-product, that there will not be efficient equilibria.[24]. However, Proposition 1 below shows that, in

[24]Indeterminancies of meaning imply inefficiency, or equivalently, efficiency implies determinancy of meaning.

the absence of language barriers, approximate efficiency can always be achieved if there are sufficiently many, albeit finite, messages.

Proposition 1 *For any $\varepsilon > 0$ and any $\langle T, (u_i)_{i\in I}\rangle$, there exists a positive integer K such that in any game $G^* = \left\langle M, T, \Lambda^*, \pi^*, A, (u_i)_{i\in I}, N^* = 1\right\rangle \in \mathcal{G}^*$, which satisfies Assumption 2, we have*

$$|M| \geq K \implies \left| \sup_{(\sigma,\rho)\in\Sigma^{G^*}} U(\sigma,\rho) - \int_{t\in T} \left[\max_{a\in A} u(t,a)\right] \pi_T(dt)\right| \leq \varepsilon,$$

where Σ^{G^*} denotes the set of equilibria of G^*.

Note that $\int_{t\in T} [\max_{a\in A} u(t,a)]\, \pi_T(dt)$ is the maximal utility that players can possibly get. We say an equilibrium (σ,ρ) achieves ε-efficiency if and only if

$$\left| U(\sigma,\rho) - \int_{t\in T} \left[\max_{a\in A} u(t,a)\right] \pi_T(dt)\right| \leq \varepsilon.$$

Then, Theorem 1 and Proposition 1 together imply the following Corollary 1. The proofs of Proposition 1 and Corollary 1 follow immediately below.[25]

Corollary 1 *Suppose Assumptions 1 and 2 hold. For any $\varepsilon > 0$ and any $\langle T, \Lambda, \pi_T, A, (u_i)_{i\in I}\rangle$, there exists a positive integer $\mathcal{N}$, such that ε-efficiency can be achieved in an equilibrium of any game $\langle M, T, \Lambda, \pi, A, (u_i)_{i\in I}, N\rangle$ in which $N \geq \mathcal{N}$ and the marginal distribution of π on T is π_T.*

Proof of Proposition 1 We use the following two lemmas to prove Proposition 1.

Lemma 7 *Suppose Assumption 2 holds. For any $\varepsilon > 0$, there exists $\delta > 0$ such that*

$$\forall t, t' \in T, d\left(t, t'\right) < \delta \implies \left|\max_{a\in A} u(t,a) - u\left[t,\ a^*\right]\right| < \varepsilon, \forall a^* \in \arg\max_{a\in A} u\left(t', a\right). \tag{33}$$

[25]Theorem 1 assumes that language types and payoff states are independently distributed. For common-interest games, however, in previous versions of this paper we showed that even in the absence of independence, with a N-dimensional protocol there exist ε-equilibria that achieve almost-efficiency.

Proof Since u is continuous and T, A are compact, u is uniformly continuous. Then, by Berge's Maximum Theorem, $\phi(t) \equiv \max_{a\in A} u(t,a)$ is continuous on $t \in T$. Since T is compact, $\phi(t)$ is uniformly continuous, and hence,

$$\forall \varepsilon > 0, \exists \delta > 0, \text{such that } d(t,t') < \delta \implies \left| \max_{a\in A} u(t,a) - \max_{a\in A} u(t',a) \right| < \frac{\varepsilon}{2}, \tag{34}$$

The uniform continuity of u implies

$$\forall \varepsilon > 0, \exists \delta > 0, \text{such that} \tag{35}$$
$$d(t,t') < \delta \implies \left| u(t,a^*) - \max_{a\in A} u(t',a) \right| = |u(t,a^*) - u(t',a^*)| < \frac{\varepsilon}{2}, \forall a^* \in \arg\max_{a\in A} u(t',a).$$

Then, (34) and (35) imply

$$\forall \varepsilon > 0, \exists \delta > 0, \text{such that } d(t,t') < \delta \implies \left| \max_{a\in A} u(t,a) - u(t,a^*) \right| < \varepsilon, \forall a^* \in \arg\max_{a\in A} u(t',a).$$

This completes the proof of Lemma 7 ∎

Lemma 8 *Suppose $|M| < \infty$. For any game $G^* = \left\langle I, M, T, \Lambda^*, \pi^*, A, (u_i)_{i\in I}, N^* \right\rangle$ satisfying Assumption 2, there exists an optimal equilibrium (σ^*, ρ^*) in G^* such that $U(\sigma^*, \rho^*) \geq U(\sigma, \rho)$ for any strategy profile (σ, ρ) in G^*.*

Proof Suppose $|M| = n$. Define a function, $\psi : A^n \longrightarrow \mathbb{R}$ as follows.

$$\psi(a_1, ..., a_n) = \int_{t\in T} \left[\max_{a\in\{a_1,...,a_n\}} u(t,a) \right] \pi_T(dt).$$

First, we show ψ is uniformly continuous, i.e.,

$$\forall \varepsilon > 0, \exists \delta > 0, \text{such that} \tag{36}$$
$$|\widehat{a}_k - \widetilde{a}_k| < \delta, \forall k \in \{1, 2, ..., n\} \implies |\psi(\widehat{a}_1, ..., \widehat{a}_n) - \psi(\widetilde{a}_1, ..., \widetilde{a}_n)| < \varepsilon.$$

By uniform continuity of u,

$$\forall \varepsilon > 0, \exists \delta > 0, \text{such that} \tag{37}$$
$$|a' - a''| < \delta \implies |[u(t,a')] - [u(t,a'')]| < \varepsilon, \forall t \in T.$$

Fix any $(\widehat{a}_1, ..., \widehat{a}_n)$ and $(\widetilde{a}_1, ..., \widetilde{a}_n)$ such that $\max_{k\in\{1,2,...,n\}} |\widehat{a}_k - \widetilde{a}_k| < \delta$. For each $t \in T$, fix any $k(t) \in \arg\max_{k\in\{1,...,n\}} u(t, \widehat{a}_k)$. We thus have

$$\psi(\widehat{a}_1, ..., \widehat{a}_n) = \int_{t\in T} \left[u\left(t, \widehat{a}_{k(t)}\right)\right] \pi_T(dt), \tag{38}$$

$$\text{and } \left|\int_{t\in T} \left[u\left(t, \widehat{a}_{k(t)}\right)\right] \pi_T(dt) - \int_{t\in T} \left[u\left(t, \widetilde{a}_{k(t)}\right)\right] \pi_T(dt)\right| < \varepsilon, \tag{39}$$

where (39) is implied by (37). Furthermore, by the definition of $\psi(\widetilde{a}_1, ..., \widetilde{a}_n)$, we have

$$\psi(\widetilde{a}_1, ..., \widetilde{a}_n) \geq \int_{t\in T} \left[u\left(t, \widetilde{a}_{k(t)}\right)\right] \pi_T(dt). \tag{40}$$

Then, (38), (39) and (40) imply

$$\begin{gathered} \forall\varepsilon > 0, \exists\delta > 0, \text{ such that} \\ |\widehat{a}_k - \widetilde{a}_k| < \delta, \forall k \in \{1, 2, ..., n\} \implies \psi(\widetilde{a}_1, ..., \widetilde{a}_n) \geq \psi(\widehat{a}_1, ..., \widehat{a}_n) - \varepsilon. \end{gathered} \tag{41}$$

If we change the roles of $(\widetilde{a}_1, ..., \widetilde{a}_n)$ and $(\widehat{a}_1, ..., \widehat{a}_n)$, and repeat the analysis, we get

$$\begin{gathered} \forall\varepsilon > 0, \exists\delta' > 0, \text{ such that} \\ |\widehat{a}_k - \widetilde{a}_k| < \delta', \forall k \in \{1, 2, ..., n\} \implies \psi(\widehat{a}_1, ..., \widehat{a}_n) \geq \psi(\widetilde{a}_1, ..., \widetilde{a}_n) - \varepsilon. \end{gathered} \tag{42}$$

Therefore, (41) and (42) imply (36), i.e., ψ is uniformly continuous.

Second, there exists

$$(a_1^*, ..., a_n^*) \in \arg\max_{(a_1,...,a_n)\in A^n} \psi(a_1, ..., a_n), \tag{43}$$

due to compactness of A and continuity of ψ, i.e.,

$$\int_{t\in T} \left[\max_{a\in\{a_1^*,...,a_n^*\}} u(t, a)\right] \pi_T(dt) \geq \int_{t\in T} \left[\max_{a\in\{a_1,...,a_n\}} u(t, a)\right] \pi_T(dt), \forall (a_1, ..., a_n) \in A^n.$$

Third, recall that there are $|M| = n$ messages. Label the elements in M as $m_1, ..., m_n$, i.e., $\{m_1, ..., m_n\}$. For any fixed strategy profile (σ, ρ), let $a_k \in A$ denote the action taken by the receiver upon getting m_k under (σ, ρ). Then, the expected utility of the players under (σ, ρ) is at most $\int_{t\in T} \left[\max_{a\in\{a_1,...,a_n\}} u(t, a)\right] \pi_T(dt)$.

Finally, $(a_1^*, ..., a_n^*)$ as defined in (43) corresponds to an equilibrium, denoted by (σ^*, ρ^*), under which players' expected utility is $\int\limits_{t \in T} \left[\max_{a \in \{a_1^*, ..., a_n^*\}} u(t, a)\right] \pi_T(dt)$. To see this, define

$$E_k = \left\{ t \in T : \; a_k^* \in \arg \max_{a \in \{a_1^*, ..., a_n^*\}} u(t, a) \right\}, \; \forall k \in \{1, 2, ..., n\}.$$

Then, define

$$\overline{E}_1 = E_1 \text{ and}$$
$$\overline{E}_k = E_k \diagdown \left[\cup_{l=1}^{k-1} E_k\right], \; \forall k \in \{2, ..., n\}.$$

As a result, $\{\overline{E}_1, ..., \overline{E}_n\}$ is a partition of T, and each a_k^* is the optimal action for every $t \in \overline{E}_k$. Thus, the following strategy profile is an equilibrium.

$$\left[\begin{array}{c} \text{sender's strategy: send } m_k \text{ if and only if } t \in \overline{E}_k, \; \forall k \in \{1, 2, ..., n\}. \\ \text{receiver's strategy: play } a_k^* \text{ if and only if he receives } m_k, \; \forall k \in \{1, 2, ..., n\}. \end{array}\right]$$

The incentive compatibility of the sender is implied by the definition of E_k and the incentive compatibility of the receiver is implied by $(a_1^*, ..., a_n^*) \in \arg\max_{(a_1, ..., a_n) \in A^n} \psi(a_1, ..., a_n)$. The last two points show the existence of an equilibrium (σ^*, ρ^*) such that $U(\sigma^*, \rho^*) \geq U(\sigma, \rho)$ for any strategy profile (σ, ρ). This completes the proof of Lemma 8 ■

We are now ready to complete the proof of Proposition 1. Fix any game satisfying Assumption 2 and any $\varepsilon > 0$. By Lemma 7, there exists $\delta > 0$ such that

$$\forall t, t' \in T, d(t, t') < \delta \implies \left| \max_{a \in A} u(t, a) - u[t, \; a^*] \right| < \varepsilon, \forall a^* \in \arg\max_{a \in A} u(t', a). \tag{44}$$

Since T is compact, it is totally bounded. Hence, there exists a positive integer K, such that T can be partitioned by $\{E_1, ..., E_K\}$ and

$$t, t' \in E_k \Longrightarrow d(t, t') < \delta, \forall k \in \{1, ..., K\}. \tag{45}$$

For each $k \in \{1, ..., K\}$, fix some $t_k \in E_k$ and some $a_k \in \arg\max_{a \in A} u(t_k, a)$. Then,

$$\begin{aligned} \sum_{k \in \{1, ..., K\}} \int_{t \in E_k} u(t, a_k) \pi_T(dt) &\geq \sum_{k \in \{1, ..., K\}} \int_{t \in E_k} \left[\max_{a \in A} u(t, a) - \varepsilon\right] \pi_T(dt) \qquad (46) \\ &= \int\limits_{t \in T} \left[\max_{a \in A} u(t, a)\right] \pi_T(dt) - \varepsilon, \end{aligned}$$

where the inequality follows from (44) and (45).

Suppose $|M| \geq K$. Then, the expected utility $\sum_{k\in\{1,...,K\}} \int_{t\in E_k} u(t, a_k)\, \pi_T(dt)$ can be achieved in a strategy profile, i.e., fix K messages, $m_1, ..., m_K$; the sender sends m_k if and only if $t \in E_k$; the receiver plays a_k if and only if he receives m_k. By Lemma 8, an optimal equilibrium exists, and denote it by (σ^*, ρ^*), and hence

$$U(\sigma^*, \rho^*) \geq \sum_{k\in\{1,...,K\}} \int_{t\in E_k} u(t, a_k)\, \pi_T(dt). \tag{47}$$

Furthermore,

$$\int_{t\in T} \left[\max_{a\in A} u(t, a)\right] \pi_T(dt) \geq U(\sigma^*, \rho^*). \tag{48}$$

Thus, (46), (47) and (48) imply

$$\left| U(\sigma^*, \rho^*) - \int_{t\in T} \left[\max_{a\in A} u(t, a)\right] \pi_T(dt) \right| \leq \varepsilon,$$

which completes the proof ■

Proof of Corollary 1 Fix any $\varepsilon > 0$ and any $\langle T, \Lambda, \pi_T, A, (u_i)_{i\in I} \rangle$. Consider any game $G = \langle M, T, \Lambda, \pi, A, (u_i)_{i\in I}, N \rangle$ satisfying Assumptions 1 and 2 and the marginal distribution of π on T is π_T. We prove Corollary 1 in 3 steps.

Step 1: by Proposition 1, there exists an equilibrium (σ^*, ρ^*) in some game $\widehat{G} = \left\langle \widehat{M}, T, \Lambda^*, \pi^*, A, (u_i)_{i\in I}, N^* = 1 \right\rangle$ such that

$$\left| U(\sigma^*, \rho^*) - \int_{t\in T} \left[\max_{a\in A} u(t, a)\right] \pi_T(dt) \right| < \varepsilon, \tag{49}$$

where $M \subset \widehat{M}$ and $\widehat{M}$ is an arbitrary set which contains sufficiently many messages. Note that $\widehat{G}$ is a game with 1-dimensional communication and no language barriers.

Step 2: by Theorem 1, a positive integer $\mathcal{N}$ exists, such that, in any game $\widetilde{G} = \left\langle \widehat{M}, T, \Lambda, \pi, A, (u_i)_{i\in I}, N \right\rangle$ similar to $\widehat{G}$ with $N \geq \mathcal{N}$, there exists an equilibrium $(\widetilde{\sigma}, \widetilde{\rho})$ of $\widetilde{G}$ that is outcome-equivalent to (σ^*, ρ^*).

Step 3: G differs from $\widetilde{G}$ only in the message set, i.e., M in G and $\widehat{M}$ in $\widetilde{G}$. In particular, the set of language types, Λ, is the same in both G and $\widetilde{G}$, i.e., $\lambda_i \subset M \subset \widehat{M}$ for every

λ_i. As a result, $(\widetilde{\sigma}, \widetilde{\rho})$ remains an equilibrium in G, if $N \geq \mathcal{N}$, because none of senders' and receivers' language types (in Λ) understand the additional messages in $\widehat{M} \diagdown M$ (i.e., these additional messages are not used in $(\widetilde{\sigma}, \widetilde{\rho})$). Finally, since $(\widetilde{\sigma}, \widetilde{\rho})$ is outcome-equivalent to (σ^*, ρ^*), (49) completes the proof. ■

B.2 Infinite-message equilibria

If we consider general communication games beyond the canonical Crawford and Sobel (1982) cheap-talk model, infinite messages may be transmitted in an equilibrium (σ^*, ρ^*) under no language barriers, i.e., $\left|\mathcal{E}^{(\sigma^*, \rho^*)}\right| = \infty$. We extend Theorem 1 to such setups. To achieve this, we need the following necessary technical assumption.

$$\left|\mathcal{E}^{(\sigma^*, \rho^*)}\right| \leq \left|\lambda_1 \bigcap \lambda_2\right|, \forall (\lambda_1, \lambda_2) \in \Lambda. \tag{50}$$

To effectively transmit equilibrium messages from λ_1 to λ_2, the sender must use the messages in $\lambda_1 \bigcap \lambda_2$ under 1-dimensional communication, and more generally, in $\left[\lambda_1 \bigcap \lambda_2\right]^N$ under N-dimensional communication, because, otherwise, λ_2 cannot understand. Furthermore, it is easy to show

$$\left|\lambda_1 \bigcap \lambda_2\right| > \infty \implies \left|\lambda_1 \bigcap \lambda_2\right| = \left|\left[\lambda_1 \bigcap \lambda_2\right]^N\right|. \tag{51}$$

Thus, to successfully mimic (σ^*, ρ^*), the set $\left[\lambda_1 \bigcap \lambda_2\right]^N$ must have (weakly) larger cardinality than $\mathcal{E}^{(\sigma^*, \rho^*)}$, which together with (51), implies the necessity of (50).

Using an argument similar to the proof of Theorem 1, we can show the following theorem.

Theorem 3 *Suppose Assumption 1 holds. For any equilibrium (σ^*, ρ^*) in any game without language barriers $G^* = \left\langle M, T, \Lambda^*, \pi^*, A, (u_i)_{i \in I}, 1 \right\rangle$ and any language barriers Λ, a positive integer $\mathcal{N}$ exists, such that in any similar game $G = \langle M, T, \Lambda, \pi, A, (u_i)_{i \in I}, N \rangle$ satisfying $N \geq \mathcal{N}$ and*

$$\left|\mathcal{E}^{(\sigma^*, \rho^*)}\right| \leq \left|\lambda_1 \bigcap \lambda_2\right|, \forall (\lambda_1, \lambda_2) \in \Lambda,$$

there exists an equilibrium (σ, ρ) of G that is outcome-equivalent to (σ^*, ρ^*).

Proof Fix any game G^* without language barriers and any equilibrium (σ^*, ρ^*) in G^*, which are listed as follows.

$$G^* = \left\langle M, T, \Lambda^*, \pi^*, A, (u_i)_{i\in I}, N^* = 1 \right\rangle;$$
$$(\sigma^*, \rho^*) = \left[(\sigma^* : T \to M)_{i\in I},\ (\rho^* : M \to A)_{i\in I} \right].$$

Given

$$\left| \mathcal{E}^{(\sigma^*, \rho^*)} \right| \leq \left| \lambda_1 \bigcap \lambda_2 \right|, \forall (\lambda_1, \lambda_2) \in \Lambda,$$

for every $(\lambda_1, \lambda_2) \in \Lambda$, there exists an injective function $\Phi_{(\lambda_1,\lambda_2)} : \mathcal{E}^{(\sigma^*, \rho^*)} \longrightarrow \lambda_1 \bigcap \lambda_2$. Consider $\mathcal{N} = \left(\overline{N} + 1\right) \times |\Lambda_2|$, where $\overline{N}$ is defined in (6). We now define a strategy profile (σ, ρ), and show it is an equilibrium in the $\mathcal{N}$-dimensional communication game which is similar to G^*.

The sender's strategy: let m_{λ_2} denote the message intended from the sender to type λ_2 of the receiver. For every $(t, \lambda_1) \in T \times \Lambda_1$, define

$$\sigma(t, \lambda_1) = (m_{\lambda_2})_{\lambda_2 \in \Lambda_2} = \left(\Upsilon_{\lambda_2}[\lambda_1], \Phi_{(\lambda_1,\lambda_2)}[\sigma^*(t)] \right)_{\lambda_2 \in \Lambda_2} \in M^{\mathcal{N}}.$$

Compared to the proof of Theorem 1 in Section 3.2.2, we replace $\Gamma_{(\lambda_1,\lambda_2)}[\sigma^*(t)]$ with $\Phi_{(\lambda_1,\lambda_2)}[\sigma^*(t)]$.

The receiver's strategy: Fix any

$$\widetilde{t} \in T.$$

Upon receiving the intended message m_{λ_2} from the sender, type λ_2 of the receiver uses the following function to translate it back to an equilibrium message under (σ^*, ρ^*) via the following function:

$$\Sigma_{\lambda_2}(m_{\lambda_2}) = \begin{cases} \sigma^*(t), & \begin{array}{l} \text{if there exists } (t, \lambda_1) \in T \times \Lambda_1 \text{ such that} \\ m_{\lambda_2} = \left(\Upsilon_{\lambda_2}[\lambda_1], \Phi_{(\lambda_1,\lambda_2)}[\sigma^*(t)] \right); \end{array} \\ \sigma^*(\widetilde{t}), & \text{otherwise.} \end{cases}$$

For any $m = (m_{\lambda_2})_{\lambda_2 \in \Lambda_2}$, where m_{λ_2} is the message from the sender to type λ_2 of the receiver, define

$$\rho(\lambda_2, m) = \rho^*\left[\Sigma_{\lambda_2}(m_{\lambda_2}) \right], \forall \lambda_2 \in \Lambda_2.$$

The rest of the argument is the same as that in the proof of Theorem 1 in Section 3.2.2 ■

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